Procyclicality of the comovement between dividend growth and consumption growth

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Abstract

Duffee (2005) shows that the amount of consumption risk (i.e., the conditional covariance between market returns and consumption growth) is procyclical. In light of this "Duffee Puzzle," I empirically demonstrate that the conditional covariance between dividend growth (i.e., the immediate cash flow part of market returns) and consumption growth is (1) procyclical and (2) a consistent source of procyclicality in the puzzle. Moreover, I solve an external habit formation model that incorporates realistic joint dynamics of dividend growth and consumption growth. The procyclical dividend-consumption comovement entails two new procyclical terms in the amount of consumption risk via cash flow and valuation channels, respectively. These two procyclical terms play an important role in generating a realistic magnitude of consumption risk. In contrast to extant habit formation models, the conditional equity premium no longer increases monotonically when a negative consumption shock arrives because it might lower the amount of risk while increasing the price of risk.

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Keywords: Duffee Puzzle, procyclical amount of consumption risk, dividend-consumption comovement, habit formation, equity premium

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1 Introduction

Duffee (2005) shows that the amount of consumption risk (i.e., the conditional covariance between market returns and consumption growth) is procyclical. This empirical finding is surprising because it makes the countercyclical equity premium harder to explain. In fact, most well-accepted consumption-based theories imply a countercyclical amount of consumption risk (e.g., Campbell and Cochrane, 1999; Bansal and Yaron, 2004; and their recent variants). Hence, I term this finding "the Duffee Puzzle."

In this paper, I decompose the amount of consumption risk into an immediate cash flow component and a valuation component, and find that the two components exhibit different cyclical behaviors. In particular, the conditional covariance between the immediate cash flow part of market returns (dividend growth) and consumption growth is procyclical and a consistent source of procyclicality in the return-consumption covariance. Moreover, I propose a parsimonious data generating process (DGP) for the joint dynamics of dividend growth and consumption growth featuring procyclical dividend-consumption comovement, and explore how realistic cash flow dynamics affect the performance of an external habit formation model.

In the empirical part of the paper, I use a dynamic conditional correlation framework to identify the cyclicalities of the amount of consumption risk and its two components given the cash flow-valuation return decomposition:

$$Cov_t\left(r^m_{t+1}, \Delta c_{t+1}\right) = \underbrace{Cov_t\left(\Delta d_{t+1}, \Delta c_{t+1}\right)}_{\text{Immediate Cash Flow}} + \underbrace{Cov_t\left(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1}\right)}_{\text{Valuation}}, \tag{1}$$

where r^m_{t+1} is the log market return, Δc_{t+1} is the log consumption growth, and Δd_{t+1} is the log dividend growth. First, I show that the total return-consumption covariance behaves weakly procyclically, despite the strongly countercyclical market return volatility and consumption growth volatility during my sample period, which adds 13 years of data to Duffee (2005) and includes the 2007-2008 financial crisis. Importantly, there is a statistically strong and positive relationship between the total return-consumption correlation and the state of the economy. For example, market returns exhibit an average correlation of 0.16 with consumption growth over the past 60 years, with the highest correlation (0.31) occurring during the expansion in early 2000 and the lowest correlation (roughly 0) occurring during the 1960-1961 recession. As for the cyclicalities of the conditional covariance components, I show that the immediate cash flow covariance behaves procyclically while the valuation covariance behaves countercyclically. Given the cash flow-valuation return decomposition, the former covariance is the only source of procyclicality in the amount of consumption risk, which sheds light on the Duffee Puzzle. This represents my core empirical finding.

The empirical part of my paper concludes with a list of eight stylized facts pertinent to my core empirical finding. These are new to the literature, except for the countercyclical conditional variances of consumption growth and market returns:

(a) The conditional variance of Δc is countercyclical.

(b) The conditional variance of Δd is procyclical.

(c) The conditional correlation between Δd and Δc is procyclical.

(d) The conditional covariance between Δd and Δc is procyclical.

(e) The conditional sensitivity (beta) of Δd to Δc is procyclical.

(f) The conditional variance of $r^m - \Delta d$ is countercyclical.

(g) The conditional variance of r^m is countercyclical.

(h) The conditional covariance between $r^m - \Delta d$ and Δc is countercyclical.

In particular, the procyclicalities of all three dividend-consumption comovement measures — correlation, covariance, and beta — are surprising findings because the asset pricing literature mostly models dividend growth and consumption growth as unit root processes with constant correlations. In the remainder of my paper, I explore how incorporating realistic joint dynamics of dividend growth and consumption growth into an endowment economy affects its equilibrium stock price dynamics.

I formulate a new DGP that matches both procyclical dividend-consumption comovement and countercyclical consumption growth volatility, which extant DGPs fail to do (e.g., Campbell and Cochrane, 1999; Bansal, Kiku, and Yaron, 2012; Segal, Shaliastovich, and Yaron, 2015; Bekaert and Engstrom, 2017). In my model, consumption receives both a "fundamental" shock and an "event" shock per period. Fundamental shocks to consumption drive contemporaneous shocks to dividends. The dividend shocks react more to fundamental shocks during booms (when past consumption growth has been high). This mechanism generates both procyclical dividend growth volatility and procyclical conditional covariance between consumption and dividends. In addition, the DGP exhibits time-varying volatility of negatively skewed event shocks to consumption. This mechanism generates countercyclical consumption growth volatility. The estimation results of the DGP reveal that the filtered fundamental shocks explain on average 82% of the total consumption growth variability, while the filtered event shocks explain as high as 34%–58% during recessions.

Finally, I solve a variant of the Campbell and Cochrane model (henceforth, CC) that accommodates the new DGP. An approximate analytical solution suggests that the procyclical dividend-consumption comovement in the new DGP entails two new *procyclical* terms in the total amount of risk: dividend risk (via cash flows) and comovement risk (via valuation). The first term is introduced by the DGP. The second term captures how pricing is affected by a persistent and procyclical dividend variance induced by dividends' procyclical exposure to the fundamental consumption shock. When a positive fundamental shock occurs in this period, future dividends are expected to react more to future consumption shocks, driving up the expected dividend growth variance. This variance becomes capitalized in stock prices, leading stock prices to react more positively to consumption shocks during booms than during recessions. The numerical solution further shows that, without procyclical dividend-consumption comovement, a CC model with countercyclical risk aversion and macroeconomic uncertainty tends to generate an amount of consumption risk that is unrealistically high. My model generates a more realistic magnitude of consumption risk, while fitting various salient asset return features in the data.

The equity premium in my model can be expressed as the product of a countercyclical price of risk (which is consistent with CC) and a time-varying amount of risk that comprises both procyclical (which is new) and countercyclical terms. These cyclical price-of-risk and amount-of-risk terms result in countervailing effects on the dynamic behavior of the conditional equity premium and the magnitude of the unconditional equity premium. The model implies a conditional equity premium that no longer increases monotonically when consumption decreases. This is because a negative fundamental consumption shock results in both higher risk aversion and lower procyclical terms in the amount of risk. In addition, the model generates a lower unconditional equity premium. This is because stock prices in my model incorporate procyclical risks, rendering the market asset less risky. The numerical solution confirms this theoretical prediction, which also yields a more realistic Sharpe Ratio.

It is noteworthy that Duffee (2005) relates this procyclical amount of consumption risk to a "composition effect": The agent's consumption growth is more correlated with market returns when financial wealth is a larger share of total wealth. My paper does not directly test this "composition effect," and does not explicitly model nonfinancial wealth. However, in equilibrium, my model does rationalize a positive relationship between the dividend-consumption comovement and the equity valuation ratio. Moreover, the comovement estimates filtered from my DGP are significantly and negatively correlated with the well-known *cay* variable from Lettau and Ludvigson (2001). Therefore, the two explanations are potentially consistent.

The outline of this paper is as follows. Section 2 examines the cyclicalities of the amount of consumption risk and its components. Section 3 formulates and estimates the new DGP. Section 4 analyzes a variant of Campbell and Cochrane's (1999) habit formation model. Concluding comments are offered in Section 5.

2 The Duffee Puzzle revisited, econometrically

The decomposition of the amount of consumption risk, as shown in Eq. (1), yields an immediate cash flow conditional covariance, $Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$, and a valuation conditional covariance, $Cov_t\left(r_{t+1}^m - \Delta d_{t+1}, \Delta c_{t+1}\right)$. In this section, I exploit a bivariate dynamic dependence model in the GARCH class to revisit the Duffee Puzzle and identify the cyclicalities of the two conditional covariances that constitute the puzzle.

2.1 The dynamic dependence model and estimation

To estimate conditional covariance, Duffee (2005) follows Campbell (1987) and Harvey (1989) and projects realizations of products of residuals on a set of instruments. This projection approach automatically enforces the identity of Eq. (1), as long as the instruments are the same on the left and right sides of the equation. However, the disadvantage is that it can only accommodate high persistence in conditional second moments by entering many lags into the set of instruments, which leads to overfitting. As a result, I follow the GARCH literature to estimate the three conditional covariances. Bivariate conditional second moments are difficult to model in the GARCH class, and the dynamic conditional correlation (DCC) framework is the state of the art. While GARCH-class models include only lagged realized second moments as instruments, my framework also accommodates business cycle instruments to help capture slow-moving cyclical behavior in conditional second moments. To achieve simplicity and to avoid concerns about nonlinearity, I use the NBER recession indicator (1=recession, 0=non-recession).

My empirical analysis uses four variables: log consumption growth Δc_{t+1}, log market return r^m_{t+1}, log dividend growth Δd_{t+1}, and the difference between market returns and dividend growth, $r^m_{t+1} - \Delta d_{t+1}$. I first project each series onto the NBER recession indicator to obtain the residual series. Denote $\boldsymbol{\widetilde{\epsilon}_{t+1}} \equiv \begin{bmatrix} \widetilde{\epsilon}_{1,t+1} & \widetilde{\epsilon}_{2,t+1} \end{bmatrix}'$ as the bivariate residual vector. Denote $h_{i,t}$ as the conditional variance of $\widetilde{\epsilon}_{i,t+1}$, $\forall i \in \{1,2\}$. I follow Engle (2002) and express the conditional variance-covariance matrix of the residuals, $\boldsymbol{H_t} \equiv E_t\left[\boldsymbol{\widetilde{\epsilon}_{t+1}\widetilde{\epsilon}'_{t+1}}\right]$, in a quadratic form:

$$\boldsymbol{H_t} = \boldsymbol{\Lambda_t Corr_t \Lambda_t}, \tag{2}$$

where $\boldsymbol{\Lambda_t}$ is a diagonal matrix with $\sqrt{h_{i,t}}$ on the diagonal, and $\boldsymbol{Corr_t}$ is a symmetric matrix with 1s on the diagonal and the model-implied conditional correlation on the off-diagonal.

Under quasi-likelihood assumptions (Bollerslev and Wooldridge, 1992; White, 1996), the log-likelihood of such a dynamic dependence model can be written as the sum of a volatility term and a correlation term. As a result, the model can be estimated by maximizing each term in separate steps. In the first step, I apply the maximum likelihood estimation (MLE) methodology to estimate the conditional variance for each residual series. Using the standardized residuals obtained from the first step, I then apply the quasi-maximum likelihood asymptotic theory to estimate the conditional correlation. The conditional variance and conditional correlation models are described in Sections 2.1.1 and 2.1.2, respectively.

2.1.1 Conditional variance

The empirical literature provides robust evidence that innovations of consumption growth and market returns are heteroskedastic (e.g., Bollerslev, Engle, and Wooldridge, 1988; Kandel and Stambaugh, 1990; Lettau, Ludvigson, and Wachter, 2008) and non-Gaussian (e.g., Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993; Bekaert and Engstrom, 2017). Given the empirical focus of this section, I choose univariate volatility specifications for consumption growth and market returns using the Bayesian information criterion (BIC) from a class of models in the GARCH literature.[1]

The choice for consumption growth Δc_{t+1} is the "GED-GARCH-q_t" form. The residual $\widetilde{\epsilon}_{t+1}$ follows a symmetric conditional generalized error distribution (GED) with an unknown shape parameter τ.[2] The conditional variance h_t has a generalized autoregressive conditional heteroskedastic process with a cyclical long-run mean $\overline{h}\,(1+q_t)$:

$$h_t = \overline{h}\,(1+q_t) + \alpha\left[\widetilde{\epsilon}_t^2 - \overline{h}\,(1+q_{t-1})\right] + \beta\left[h_{t-1} - \overline{h}\,(1+q_{t-1})\right], \tag{3}$$

$$q_t = \nu SNBER_t, \tag{4}$$

where $\overline{h}$ denotes the predetermined unconditional variance; $\alpha+\beta<1$, and $\alpha,\beta>0$. The q_t process is modeled as a multiple of the standardized NBER recession indicator, denoted by $SNBER_t$,[3] so that the average conditional variance $E(h_t)$ is $\overline{h}$; ν is an unknown parameter. The sign of ν identifies cyclicality within the model. A positive (negative) ν indicates that the long-run mean of conditional variance is countercyclical (procyclical), and a zero ν fails to reject the null of a constant long-run mean. The GED-GARCH model (Nelson, 1991) is a special case of $\nu=0$.

The choice for market returns r^m_{t+1} is the "BEGE-GARCH-q_t" form, a variant of the "Bad Environment-Good Environment" (BEGE) model by Bekaert, Engstrom, and Ermolov (2015). The residual follows a composite distribution of two centered gamma shocks that independently govern the dynamics of the two tails. The composite residual follows $\widetilde{\epsilon}_{t+1} = \sigma_{hp}\widetilde{\omega}_{hp,t+1} - \sigma_{hn}\widetilde{\omega}_{hn,t+1}$, where $\widetilde{\omega}_{hp,t+1} \sim \Gamma(hp_t,1) - hp_t \equiv \widetilde{\Gamma}(hp_t,1)$, $\widetilde{\omega}_{hn,t+1} \sim \Gamma(hn_t,1) - hn_t \equiv \widetilde{\Gamma}(hn_t,1)$, shape parameters $hp_t>0$ and $hn_t>0$ for all t, and scale parameters $\sigma_{hp}>0$ and $\sigma_{hn}>0$. The left- and right-tail shape parameters — hn_t and hp_t — are modeled as follows:

$$hn_t = \overline{hn}\,(1+q_t) + \alpha_{hn}\left[\frac{\widetilde{\epsilon}_t^2}{2\sigma_{hn}^2} - \overline{hn}\,(1+q_{t-1})\right] + \beta_{hn}\left[hn_{t-1} - \overline{hn}\,(1+q_{t-1})\right], \tag{5}$$

$$hp_t = \overline{hp}\,(1+q_t) + \alpha_{hp}\left[\frac{\widetilde{\epsilon}_t^2}{2\sigma_{hp}^2} - \overline{hp}\,(1+q_{t-1})\right] + \beta_{hp}\left[hp_{t-1} - \overline{hp}\,(1+q_{t-1})\right], \tag{6}$$

[1] Table IA1 of the Internet Appendix provides the detailed model selection results for consumption growth and market returns.

[2] $\tau <(>)\ 2$ indicates that this GED distribution exhibits heavier (lighter) tails than the Gaussian distribution.

[3] The value of $SNBER$ is -0.4029 in non-recession months and 2.4782 in recession months.

where $\overline{hn}$ and $\overline{hp}$ are unknown parameters because they depend on the values of scale parameters σ_{hn} and σ_{hp}, respectively; $\alpha_{hn} + \beta_{hn} < 1$, $\alpha_{hp} + \beta_{hp} < 1$, and $\alpha_{hn}, \alpha_{hp}, \beta_{hn}, \beta_{hp} > 0$; and q_t is modeled as in Eq. (4). The implied total conditional variance h_t is $\sigma_{hp}^2 hp_t + \sigma_{hn}^2 hn_t$. Similarly, a positive (negative) ν indicates countercyclical (procyclical) total return variance.

For the purpose of consistency, I choose conditional variance models for market return components from the BEGE-GARCH framework. In addition to the full BEGE model (above), I also consider two half BEGE models, "BEGE-hn_t-GARCH-q_t" and "BEGE-hp_t-GARCH-q_t", that allow heteroskedasticity from the left- and right-tail gamma shocks, respectively. Consistent with the weakly positive skewness of dividend growth Δd_{t+1} and the strongly negative skewness of the valuation part of market returns $r^m_{t+1} - \Delta d_{t+1}$, the choice for Δd_{t+1} is the BEGE-hp_t-GARCH-q_t model, and the choice for $r^m_{t+1} - \Delta d_{t+1}$ is the BEGE-hn_t-GARCH-q_t model. These choices reveal that dividend growth (valuation) is important in explaining the right-tail (left-tail) heteroskedasticity of market returns.[4]

2.1.2 Conditional correlation

The conditional correlation matrix $\boldsymbol{Corr_t}$ from Eq. (2) is written as $\left(\boldsymbol{Q_t^*}\right)^{-1} \boldsymbol{Q_t} \left(\boldsymbol{Q_t^*}\right)^{-1}$, where $\boldsymbol{Q_t^*}$ is the diagonal matrix with the square root of the diagonal element of $\boldsymbol{Q_t}$ on the diagonal so that the diagonal entries of $\boldsymbol{Corr_t}$ are 1s. Denote $\boldsymbol{z_{t+1}} \equiv \begin{bmatrix} z_{1,t+1} & z_{2,t+1} \end{bmatrix}' = \boldsymbol{\Lambda_t^{-1}} \boldsymbol{\widetilde{\epsilon}_{t+1}}$ as the standardized residuals. The key dynamic conditional correlation process is modeled as follows:

$$\boldsymbol{Q_t} = \overline{Q}_{12} \begin{bmatrix} 1 & 1+q_t \\ 1+q_t & 1 \end{bmatrix} + \alpha_{12} \left[\boldsymbol{z_t z_t'} - \overline{Q}_{12} \begin{bmatrix} 1 & 1+q_{t-1} \\ 1+q_{t-1} & 1 \end{bmatrix} \right]$$

[4]Table IA2 of the Internet Appendix provides the detailed model selection results for market return components. I detail three additional technical notes below to further explain the full and half BEGE models in this section. First, one q_t process for both hp_t and hn_t is suitable in fitting market return conditional variance because both shape parameters hp_t and hn_t are countercyclical in data. That is, there are higher chances of observing extreme return residual realizations from both tails during recessions. One can consider a full BEGE model with two q_t processes and two ν estimates associated with the two tails, respectively. However, a full BEGE model with two q_t processes exhibits two disadvantages: (1) more parameters mean a more complex estimation system with slower convergence; (2) given the slow-moving nature of q_t, it might be empirically difficult to separate the long-run mean of the total conditional variance into two (tail) processes. As a result, there may be little advantage in allowing two q_t processes when estimating the conditional variance of market returns. Second, centered left-tail (right-tail) gamma shocks have positive densities for both negative and positive realizations, albeit bounded above (below). As a result, a "half" BEGE model does not mean that it allows heteroskedasticity from only positive or only negative realizations. Third, the conditional variance of BEGE-hp_t-GARCH-q_t is $h_t = \sigma_{hp} hp_t + \sigma_{hn} \overline{hn}$; the conditional variance of BEGE-hn_t-GARCH-q_t is $h_t = \sigma_{hp} \overline{hp} + \sigma_{hn} hn_t$.

$$+ \beta_{12} \left[\boldsymbol{Q_{t-1}} - \overline{Q}_{12} \begin{bmatrix} 1 & 1+q_{t-1} \\ 1+q_{t-1} & 1 \end{bmatrix} \right], \quad (7)$$

where $\overline{Q}_{12}$ denotes the predetermined constant correlation of the standardized residuals, q_t is modeled as in Eq. (4), and standard stationarity assumptions apply. I refer to this model as "DCC-q_t" in this paper. It is noteworthy that Engle (2002) assumes a constant long-run mean. Meanwhile, Colacito, Engle, and Ghysels (2011) use a weighted average of past correlations to model the long-run conditional mean. Unlike these models, the present DCC model directly links time-varying correlation to the state of the economy through an instrumented process q_t.

2.2 Data

I follow Duffee (2005) to use monthly data indexed with t. Monthly real consumption is defined as the sum of seasonally adjusted real aggregate expenditures on nondurable goods and services (source: U.S. Bureau of Economic Analysis, BEA). The deflators for aggregate nondurable goods and services consumption are different (source: BEA). Monthly dividends are measured by the real 12-month trailing dividends of the NYSE/AMEX/NASDAQ universe (source: Center for Research in Security Prices, CRSP), allowing for reinvestment at the gross risk free rates (source: CRSP). Inflation is calculated using the CPI (source: Federal Reserve Economic Data, FRED). Monthly consumption (dividend) growth is defined as log-differenced real consumption (dividend) per capita. The monthly population is obtained from the BEA. Monthly real market returns are the log value-weighted market return including dividends (source: CRSP, NYSE/AMEX/NASDAQ), minus inflation. The sample spans the period between January 1959 and June 2014.

It is well-known that measured aggregate consumption data are flow data that are reported as total consumption over an extended period. This temporal aggregation results in a non-zero autoregressive coefficient of aggregate consumption growth (Working, 1960), even if the true consumption growth is i.i.d. The temporal aggregation effect could also lead to biases in the estimated conditional covariances, as thoroughly discussed in Duffee (2005, pp. 1691-1694). Therefore, I follow the literature and construct a measure of monthly consumption growth that removes the autoregressive terms up to the third order, $\Delta c_{t+1} - \sum_{i=1}^{3} \phi_i (\Delta c_{t+1-i} - \overline{c})$, where ϕ_i is the i^{th}-order autoregressive coefficient and $\overline{c}$ is the sample mean. For the rest of this paper, "consumption growth" refers to this measure that controls for temporal aggregation.

The present research uses observed dividend data to identify stylized facts (this section) and provide exact data point estimates to be matched by a more general economic model (later). As a result, a one-time dividend event — unique and anticipated never to recur — should be excluded from the analysis because such an event is not drawn from the distribution relevant to current and future prices. In my sample period, I identify two extremely large dividend payments that significantly inflated dividend growth volatility and are considered unrepresentative: (1) The Microsoft special dividend payment in November 2004, and (2) the expiration of the Jobs and Growth Tax Relief Reconciliation Act of 2003 (known as the "The Bush Tax Cuts") on December 31, 2012, which incentivized a special dividend uptick during Q4 of 2012. I provide two observations that support this identification. First, while the 99th percentile of the real 12-month trailing dividend growth distribution is 3%, monthly dividend growth rates during November 2004 and December 2012 are 13% and 7%, respectively. Second, due to the 12-month trailing calculation, these two events result in the two lowest monthly dividend growth rates 12 months later: -7% in November 2005 and -4% in December 2013. Meanwhile, the 1st percentile of the distribution is -2.5%. To treat these two events, I linearly interpolate the corresponding CRSP-implied dividend data points using values before and after, prior to the 12-month trailing calculation.

2.3 Results

2.3.1 The total conditional covariance

In this section, I discuss the core object of interest: the cyclicality of the conditional covariance between market returns and consumption growth. The dynamic dependence model implies three sources of cyclicality: consumption growth volatility, return volatility, and their correlation. As shown in Panel A of Table 1, the estimated cyclicality parameters (ν) in both consumption growth and market return conditional variance models are significant and positive, suggesting countercyclical conditional volatilities. This result is consistent with Bollerslev, Engle, and Wooldridge (1988), Schwert (1989), Kandel and Stambaugh (1990), and Hamilton and Lin (1996), among many others. In terms of economic magnitude, Panel B shows that the average conditional volatility of consumption growth is 0.0030 conditioned on "No Recession" periods and 0.0032 conditioned on "Recession" periods; the t-statistic of their difference is 2.18. Similarly, the average return conditional volatility is 0.0406 during non-recession periods and 0.0579 during recession periods (t-statistic=37.94).

Table 1 also provides strong evidence for a procyclical return-consumption conditional correlation. The significant and negative cyclicality parameter estimate in Panel A (ν=-0.1153, SE=0.0339) implies an average conditional correlation of 0.1652 during non-recession periods and 0.1128 during recession periods.[5] Moreover, Panel B presents the fitted conditional correlation averages in these two states of the economy, 0.1651 and 0.1099, respectively, and supports a significantly lower average during recession periods (t-statistic=-9.66). It can be shown that the model-implied averages are statistically close to the fitted averages.

The product of the three conditional second moment estimates yields the return-consumption conditional covariance estimates. From Panel B of Table 1, the conditional covariance dynamics appear weakly procyclical. This is expected because both the strongly countercyclical return and consumption growth volatilities dampen their procyclical correlation. The implied conditional sensitivity (beta) of market returns to consumption growth, measured as the ratio of the conditional covariance to the consumption growth conditional variance, also exhibits weakly procyclical behavior.

It is noteworthy that my empirical analysis uses the NBER recession indicator as the business cycle instrument (as motivated in Section 2.1), which is different from Duffee's (2005) main instrument, the wealth-consumption ratio. As a robustness check, I examine how my empirical estimates of return-consumption conditional comovement (correlation, covariance, and beta) vary with the de-trended $\widehat{cay}$ from Lettau and Ludvigson (2001). Consistent with Duffee (2005), all three comovement estimates are significantly and negatively correlated with $\widehat{cay}$; Table IA3 of the Internet Appendix provides more details.

2.3.2 The decomposition

In this section, I decompose the total return conditional covariance into an immediate cash flow component and a valuation component, and examine the cyclicalities of all second

[5]The long-run mean is modeled as $\overline{Q}_{12}(1 + q_t) = \overline{Q}_{12}(1 + \nu \times SNBER_t)$. Non-recession long-run mean calculation: 0.1652 = 0.1579×(1+(-0.1153×-0.4029)), where -0.4029 is the non-recession value of the standardized NBER recession indicator denoted by $SNBER_t$ (as introduced in Section 2.1) and 0.1579 is the predetermined correlation of standardized residuals $\overline{Q}_{12}$ (as shown in Table 1); recession long-run mean calculation: 0.1128 = 0.1579×(1+(-0.1153×2.4782)), where 2.4782 is the recession value of $SNBER_t$.

moments pertinent to this decomposition. First, the conditional variances of the market return components exhibit different cyclical behaviors. The dividend growth conditional variance is found to be procyclical given the negative cyclicality parameter estimate shown in Table 2. The average conditional volatility of dividend growth during recessions, 0.0098, is statistically smaller than that during booms, 0.0118, with a t-statistic of -8.02. On the other hand, both the positive cyclicality parameter estimate (from Panel A) and the significantly higher recession-period conditional volatility average (from Panel B) constitute strong evidence for a countercyclical valuation conditional variance.

Next, I focus on the two comovement estimation results. As shown in Table 2, the cyclicality parameter in the dividend-consumption conditional correlation model is estimated to be significant and negative (ν=-0.1896, SE=0.1076), suggesting a procyclical conditional correlation process. The conditional correlation fluctuates around 0.0152 during non-recession periods and around 0.0075 during recession periods.[6][7] The model-implied averages are statistically close to their empirical counterparts at the 5% significance level, as shown in Panel B. Moreover, the dividend-consumption conditional covariance and beta also appear to be significantly procyclical. All of the four procyclicality findings above — dividend-consumption correlation, covariance, beta, and dividend volatility — are new to the literature.

The conditional correlation between the valuation part of market returns and consumption growth behaves procyclically given the negative cyclicality parameter estimate (ν=-0.1088, SE=0.0328). The implied non-recession average is 0.1554, while the recession average is 0.1088.[8] Similar averages are shown using empirical estimates, per Panel B. Importantly, the strongly

[6] The long-run mean is modeled as $\overline{Q}_{12}(1 + q_t) = \overline{Q}_{12}(1 + \nu \times SNBER_t)$. Non-recession long-run mean calculation: 0.0152 = 0.0141×(1+(-0.1896×-0.4029)), where -0.4029 is the non-recession value of $SNBER_t$ and 0.0141 is $\overline{Q}_{12}$ (as shown in Table 2); recession long-run mean calculation: 0.0075 = 0.0141×(1+(-0.1896×2.4782)), where 2.4782 is the recession value of $SNBER_t$.

[7] At the monthly frequency, the unconditional correlation between dividend growth and consumption growth is 0.0569 using the raw data, 0.0242 using residuals, and 0.0141 using standardized residuals; see more details in Table IA4 of the Internet Appendix. The quarterly unconditional correlation between dividend growth and consumption growth (i.e., quarterly growth rate=the sum of monthly growth rates within the same quarter) in my sample is 0.1568. In an earlier version of the paper, I considered Shiller's monthly aggregate dividend data. One can replicate his data in two steps: (1) obtain the 12-month trailing CRSP-implied dividend levels at the quarterly frequency, and (2) calculate dividends for other months using linear interpolation, e.g., April = $\frac{2}{3}$March + $\frac{1}{3}$June and May = $\frac{1}{3}$March + $\frac{2}{3}$June, and so on. Due to this linear interpolation methodology, Shiller's monthly dividend growth series (essentially capturing information at the quarterly frequency) is smoothed and has an extremely high correlation of 0.18 with the monthly consumption growth. Hence, his monthly dividend data are not suitable in the present research.

[8] The long-run mean is modeled as $\overline{Q}_{12}(1 + q_t) = \overline{Q}_{12}(1 + \nu \times SNBER_t)$. Non-recession long-run mean calculation: 0.1554 = 0.1489×(1+(-0.1088×-0.4029)), where -0.4029 is the non-recession value of $SNBER_t$ and 0.1489 is $\overline{Q}_{12}$ (as shown in Table 2); recession long-run mean calculation: 0.1088 = 0.1489×(1+(-0.1088×2.4782)), where 2.4782 is the recession value of $SNBER_t$.

countercyclical consumption and valuation volatilities (discussed above) dampen this procyclical correlation, resulting in an overall weakly countercyclical valuation covariance.

In summary, I provide evidence for a procyclical return-consumption conditional covariance, even after adopting sophisticated conditional variance models (which account for heteroskedasticity, non-Gaussianity, and asymmetry) and using a long sample period (which includes the 2007-2008 financial crisis). Moreover, I show that the conditional covariance between dividend growth and consumption growth is a unique and consistent source of procyclicality in the total return-consumption conditional covariance, given the current return decomposition. This is the core empirical contribution of my paper.

Although inspired by the identity of Eq. (1), this decomposition analysis does not enforce its mathematical equality in the estimation in order to gain flexibility with the model. As a validation check, Table 3 conducts two closeness tests comparing the direct (left side of Eq. (1)) and the indirect (right side) estimates of return-consumption conditional correlation, covariance, and beta. Panel A is a correlation test; the regression coefficients of the indirect return-consumption conditional comovement estimates on the direct estimates are all statistically close to 1 at the 5% significance level. Panel B is a series of equality tests using key sample moments, including mean, standard deviation, and scaled skewness. Sample moments calculated using the direct estimates are all within 1.96 standard deviations from the indirect point estimates. Hence, the two statistical tests support the empirical identity of Eq. (1).

2.3.3 Time series

Although conditional covariances play a central role in asset pricing, conditional correlations are more intuitive and easier to interpret and illustrate. Fig. 1 shows the dynamics of the estimated return-consumption conditional correlation. First, there exists a wide time variation around its sample mean of 0.16 over the past 60 years, reaching lows of (roughly) 0 during the 1960 recession and the 2007-2008 recession and a high of 0.31 in early 2000. Hump-shaped patterns are evident during the 1960s, 1970s, and 1980s expansions. Second, the return-consumption conditional correlation appears to be persistent, which is consistent with its AR(1) coefficient estimate of 0.95 from Table 1. Furthermore, incorporating the dynamics of the conditional variances, the implied conditional covariance (beta) exhibits a similar pattern to the conditional correlation, but tends to be higher before (after) the 1990s. This is because the conditional volatility of consumption growth has been consistently found to be lower after 1992.[9]

In implementing the identity of Eq. (1), the return-consumption conditional correlation $Corr_t(r^m_{t+1}, \Delta c_{t+1})$ can be expressed as follows:

$$\begin{aligned} Corr_t(r^m_{t+1}, \Delta c_{t+1}) = &\frac{\sigma_t(\Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(\Delta d_{t+1}, \Delta c_{t+1}) \\ &+ \frac{\sigma_t(r^m_{t+1} - \Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1}). \end{aligned} \tag{8}$$

Fig. 2 depicts the dynamics of these two components in the correlation space.[10] In particular, the immediate cash flow component is smaller in magnitude and less persistent. Its highest value (0.10) appeared in April 1984, which coincided with the largest drop in monthly real consumption during the sample period (-12.90 annualized percents[11]) and a large dividend-consumption correlation of 0.25. To contextualize this magnitude (0.10), the immediate cash flow component accounted for 0.10 of the total return-consumption correlation (0.24) in that month, or a contribution of 41.67%. There is evidence that the immediate cash flow component, on average, contributes more to the total return-consumption correlation during booms (5.63%) than during recessions (1.95%). Notably, the three highest contributions were 52.98% in the late 1980 expansion, 53.63% in the 2009 expansion, and 45.70% in the 2013 expansion.

2.4 Summary

I conclude the empirical part of the paper with a list of stylized facts on the cyclicalities of the relevant conditional second moments:

[9] Figure IA1 of the Internet Appendix provides the time series plot of the consumption growth conditional volatility.

[10] Fig. 2 uses the difference between the estimates of $Corr_t(r^m_{t+1}, \Delta c_{t+1})$ and the estimates of (a) $\frac{\sigma_t(\Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$ as the proxy for the non-cash flow component, or (b) above. Hence, this difference is an indirect measure of the non-cash flow component. Figure IA2 of the Internet Appendix compares the indirect and direct measures of the non-cash flow component, and shows that these two time series are correlated at 0.99.

[11] In early 1984, the nominal non-durable consumption level decreased significantly while its price index increased, which together resulted in the largest drop in real consumption growth during the sample period and a large increase in consumption volatility. Figure IA1 of the Internet Appendix reports the time series plots of conditional volatilities of consumption growth and dividend growth.

(a)	The conditional variance of Δc is countercyclical.	*Kandel and Stambaugh (1990)*
(b)	The conditional variance of Δd is procyclical.	*New*
(c)	The conditional correlation between Δd and Δc is procyclical.	*New*
(d)	The conditional covariance between Δd and Δc is procyclical.	*New*
(e)	The conditional sensitivity (beta) of Δd to Δc is procyclical.	*New*
(f)	The conditional variance of $r^m - \Delta d$ is countercyclical.	*New*
(g)	The conditional variance of r^m is countercyclical.	*Schwert (1989)*
(h)	The conditional covariance between $r^m - \Delta d$ and Δc is countercyclical.	*New*

3 A new DGP for the joint dividend-consumption dynamics

Extant consumption-based asset pricing theories typically do not discuss whether the modeling choice of aggregate dividends is realistic or not. In a Lucas tree economy (Lucas, 1978), dividends equal consumption, but the literature mostly models them as unit root processes with constant correlations.[12] Table 4 summarizes seven consumption-based asset pricing models and their abilities to match the empirical facts established in Section 2. Using suggested parameter choices, DGPs of these models fail to generate realistic dynamics of dividend-consumption comovement and dividend variance. Furthermore, these models imply either a countercyclical or zero return-consumption covariance, which contradicts the Duffee Puzzle.

This section presents a new DGP that has the potential to accommodate Facts (a)–(e) into a consumption-based asset pricing model with a minimum number of state variables. To enhance the plausibility of the new DGP, I discuss the economic interpretations of the state variables and shocks given the estimation results. In Appendix Appendix A, I discuss why alternative modeling approaches may be less suited to fit the salient empirical facts.

3.1 The DGP

The consumption growth, Δc_{t+1}, is assumed with a constant mean $\overline{c}$ and a composite shock structure:

$$\Delta c_{t+1} = \overline{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, \tag{9}$$

[12]For example, Campbell and Cochrane (1999) assume constant comovement and variances. Bansal and Yaron (2004) assume a zero dividend-consumption comovement given their shock assumption. Bansal, Kiku, and Yaron (2012) allow dividend growth to have a constant exposure to the consumption shock.

$$\widetilde{\omega}_{c,t+1} \sim N(0,1),$$
$$\widetilde{\omega}_{n,t+1} \sim \Gamma(n_t, 1) - n_t.$$

The "fundamental" consumption shock, $\widetilde{\omega}_{c,t+1}$, is a Gaussian shock with unit standard deviation, and its associated scale parameter σ_c is positive. The "event" consumption shock, $\widetilde{\omega}_{n,t+1}$, follows a centered, positively-skewed, and heteroskedastic gamma distribution with a strictly positive shape parameter n_t. To realistically capture negative skewness in consumption growth, scale parameter σ_n is negative. Given the moment generating functions of Gaussian and gamma shocks, the conditional variance of Δc_{t+1}, denoted by $V_{c,t}$, is a linear function of n_t: $V_{c,t} = \sigma_c^2 + \sigma_n^2 n_t$. As a result, n_t is referred to as the macroeconomic uncertainty state variable in this DGP.

The dividend growth, Δd_{t+1}, has the following process:

$$\Delta d_{t+1} = \overline{d} + \phi_d \left(V_{c,t} - \overline{V}_c\right) + b_t \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, \tag{10}$$
$$\widetilde{\omega}_{d,t+1} \sim \Gamma(V_d, 1) - V_d.$$

The expected dividend growth has a constant part and a time-varying part that decreases with macroeconomic uncertainty ($\phi_d < 0$); $\overline{V}_c$ denotes the mean of $V_{c,t}$. State variable b_t captures the time-varying sensitivity of dividend growth to consumption growth through the fundamental shock. The dividend-specific shock, $\widetilde{\omega}_{d,t+1}$, follows a centered, positively-skewed, and homoskedastic gamma distribution with a strictly positive shape parameter V_d. Scale parameter σ_d is positive to capture positive dividend growth skewness.[13]

The three shocks are mutually independent. Δc_{t+1} and Δd_{t+1} are observables. The two latent state variables, n_t and b_t, follow autoregressive processes that have positive exposures to the event and fundamental shocks, respectively (ϕ_n, ϕ_b, σ_{nn}, $\lambda_b > 0$):

$$n_{t+1} = (1 - \phi_n)\overline{n} + \phi_n n_t + \sigma_{nn} \widetilde{\omega}_{n,t+1}, \tag{11}$$
$$b_{t+1} = (1 - \phi_b)\overline{b} + \phi_b b_t + \lambda_b \sigma_c \widetilde{\omega}_{c,t+1}. \tag{12}$$

[13]The conditional unscaled skewness $E_t\left[(\Delta d_{t+1} - E_t(\Delta d_{t+1}))^3\right]$ is $2\sigma_d^3 V_d$. Proof: $E_t\left[(\Delta d_{t+1} - E_t(\Delta d_{t+1}))^3\right] = E_t\left[(b_t\sigma_c\widetilde{\omega}_{c,t+1} + \sigma_d\widetilde{\omega}_{d,t+1})^3\right] = E_t\left[(b_t\sigma_c\widetilde{\omega}_{c,t+1})^3 + (\sigma_d\widetilde{\omega}_{d,t+1})^3 + 3(b_t\sigma_c\widetilde{\omega}_{c,t+1})^2\sigma_d\widetilde{\omega}_{d,t+1} + 3b_t\sigma_c\widetilde{\omega}_{c,t+1}(\sigma_d\widetilde{\omega}_{d,t+1})^2\right] = E_t\left[0 + (\sigma_d\widetilde{\omega}_{d,t+1})^3 + 0 + 0\right] = E_t\left[(\sigma_d\widetilde{\omega}_{d,t+1})^3\right] = 2\sigma_d^3 V_d$.

The key reason for this DGP to imply both countercyclical consumption growth variance and procyclical dividend growth variance and comovement is the flexible use of the two consumption shocks. The DGP exhibits time-varying volatility of consumption growth through *event* consumption shocks. Meanwhile, dividends react more to *fundamental* consumption shocks in booms (times when past consumption growth has been high). The cyclicalities of the two state variables can be proved analytically:

■ *Fact Check (a): n_{t+1} is countercyclical, given $Cov_t(\Delta c_{t+1}, n_{t+1}) = \sigma_n \sigma_{nn} n_t < 0$.*

■ *Fact Check (e): b_{t+1} is procyclical, given $Cov_t(\Delta c_{t+1}, b_{t+1}) = \lambda_b \sigma_c^2 > 0$.*

The procyclicalities of dividend growth variance and dividend-consumption comovement follow:

■ *Fact Check (b): The conditional variance of dividend growth, $b_t^2 \sigma_c^2 + \sigma_d^2 V_d$, is procyclical if $b_t > \frac{(\phi_b - 1)\bar{b}}{\phi_b}$. (Note: $\frac{(\phi_b - 1)\bar{b}}{\phi_b} < 0$)*

■ *Fact Check (c): The conditional correlation between dividend and consumption growth, $\frac{b_t \sigma_c^2}{\sqrt{\sigma_c^2 + \sigma_d^2 n_t}\sqrt{b_t^2 \sigma_c^2 + \sigma_d^2 V_d}}$, is procyclical given a countercyclical n_t and a procyclical b_t.*

■ *Fact Check (d): The conditional covariance between dividend and consumption growth, $b_t \sigma_c^2$, is procyclical.*

3.2 DGP estimation results

Given that there is no feedback from the cash flow process to consumption, I estimate the consumption growth system $\{\Delta c_t, n_t\}_{t=1}^T$ and the dividend growth system $\{\Delta d_t, b_t\}_{t=1}^T$ in two separate steps using MLE-type methodologies. Appendix Appendix B details the estimation procedure.

The estimation results in Table 5 confirm that the new DGP matches Facts (a)–(e). Confirming Fact (a), consumption growth depends negatively on the positively-skewed heteroskedastic event shock given the estimate of σ_n, while the macroeconomic uncertainty state variable n_t has a positive exposure to the event shock given the estimate of σ_{nn}. Confirming Fact (e), the estimate of λ_b suggests that the new comovement state variable b_t loads significantly and positively on the fundamental consumption shock, rendering b_t a procyclical variable. To provide direct evidence for Facts (a) and (e), the regression coefficients of n_t and b_t on the NBER recession indicator are 0.5926 (SE=0.0354) and -0.1240 (SE=0.0190), respectively. Similarly, confirming Facts (b)–(d), the DGP-implied dividend growth conditional variance, dividend-consumption conditional correlation, and conditional covariance are all procyclical given their significant and negative NBER loadings.

Fig. 3 depicts the time variation in the monthly estimates of the two DGP state variables: macroeconomic uncertainty n_t (top) and dividend-consumption comovement b_t (bottom). The monthly n_t estimates exhibit a persistent process that occasionally spikes, mostly during recessions. Its countercyclical nature determines the countercyclicality of the conditional variance of consumption growth, $\sigma_c^2+\sigma_n^2 n_t$. Given the estimation, while σ_c^2 contributed by the fundamental shock explains on average 82% of the total consumption growth variance, $\sigma_n^2 n_t$ contributed by the event shock could explain as high as 34%–58% during recessions. On the other hand, as shown in the bottom plot of Fig. 3, the dividend-consumption comovement state variable b_t is a less persistent process, which is consistent with observations from the empirical model in Section 2. The monthly b_t estimates exhibit a significant and negative correlation of -0.25 with the NBER recession indicator.

3.3 Economic interpretation of consumption shocks

The consumption shock structure plays a crucial role in enabling this DGP to simultaneously satisfy Facts (a)–(e). In the following section, I motivate the economic interpretations of the two consumption shocks.

First, the filtered fundamental consumption shock $\widetilde{\omega}_c$ is procyclical, given its significant and negative correlation with the NBER recession indicator at various frequencies (monthly: -0.18; quarterly: -0.27; Panel A of Table 6). From the top plot of Fig. 4, negative spikes in the filtered $\widetilde{\omega}_c$ often appear during recessions. Moreover, the fundamental shock also comoves significantly and negatively with the de-trended quarterly consumption-wealth ratio $\widehat{cay}$ from Lettau and Ludvigson (2001), given evidence from Table 6 and Fig. 4. This negative relationship is consistent with the DGP. In this DGP, a unit fundamental shock at time t increases b_t and has persistent and positive effects on the expected future dividend-consumption comovement and dividend variance. This variance becomes capitalized in financial wealth, inducing a higher asset price. Finally, to justify the statistical assumptions about the fundamental shock, Panel B of Table 6 shows that the filtered fundamental shock is statistically close to a standardized Gaussian shock.

Second, as shown in the bottom plot of Fig. 4, major positive spikes of the filtered event consumption shock $\widetilde{\omega}_n$ occur during recessions. Because consumption growth loads negatively on the event shock, these spikes reflect extreme negative consumption growth events. Table 6 formally confirms the countercyclicality of $\widetilde{\omega}_n$, as evidenced by its correlation of 0.13 (0.25) with the monthly (quarterly) NBER recession indicator. In addition, $\widehat{cay}$ appears uncorrelated with the filtered event shock $\widetilde{\omega}_n$ and dividend-specific shock $\widetilde{\omega}_d$. This result is consistent with the theory, while in turn supports the possibly close economic relationship between $\widehat{cay}$ and the fundamental shock, as discussed above.

It is noteworthy that several recent models in the consumption-based asset pricing literature have attempted to model consumption growth innovation with two independent shocks. The continuous-time model in Longstaff and Piazzesi (2004) models the consumption growth innovation with a Brownian motion and a jump process — which are conceptually similar to the Gaussian fundamental shock and the gamma event shock in my DGP. Closer to my model, the DGP in Bekaert and Engstrom (2017) features two independent heteroskedastic gamma shocks, one associated with the "good" volatility and the other with "bad" volatility. In a similar vein, Segal, Shaliastovich, and Yaron (2015) explore good and bad shocks in a long-run risk framework. However, none of these models accommodate realistic dividend-consumption comovement.

4 An external habit model

The theoretical model in this section explores how incorporating realistic joint dynamics of dividend growth and consumption growth into an endowment economy affects equilibrium stock price dynamics, thus potentially accommodating the Duffee Puzzle. Between the two puzzle components, the procyclical cash flow conditional covariance is immediately satisfied given the new DGP. However, different consumption-based asset pricing paradigms have different implications for the cyclicality of the valuation component of the puzzle. In particular, the external habit formation paradigm is suitable to structurally examine the Duffee Puzzle for the following reasons. First, Campbell and Cochrane (1999; CC) naturally entails a countercyclical valuation covariance through risk aversion: the effect of consumption shocks on equity valuation ratios is amplified when risk aversion is higher. Second, as I show later, an endowment economy with procyclical dividend risk requires a countercyclical price of risk to generate realistic, procyclical equity prices. Third, this paradigm implies an equity premium that equals the product of time-varying price of risk and amount of risk, which is consistent with Duffee's (2005) original theoretical motivation for studying market return covariance.

Section 4.1 introduces the model. Section 4.2 derives (approximate) analytical model solution and implications. Section 4.3 tests the numerical model solution with a wide range of empirical moments, featuring the eight stylized facts established in Section 2.

4.1 Pricing kernel, risk free rate, sensitivity function

I obtain the log real pricing kernel using the external habit preference as in the CC model:

$$m_{t+1} = \ln \beta - \gamma \Delta c_{t+1} - \gamma \Delta s_{t+1}, \tag{13}$$

where β is the time discount factor, γ is the curvature parameter, and Δs_{t+1} is the change in the log surplus consumption ratio, $s_{t+1} - s_t$. The dynamics of the log surplus consumption ratio incorporate the new consumption growth innovation:

$$s_{t+1} = (1 - \phi_s)\overline{s}_t + \phi_s s_t + \lambda_t \left(\sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1} \right), \tag{14}$$

where ϕ_s is the persistence coefficient, $\overline{s}_t$ is the time-varying long-run mean, and λ_t is the sensitivity function.

The real risk free rate, rf_t, is solved from the first-order condition for the consumption-saving choice, $rf_t = \ln E_t[\exp(m_{t+1})]^{-1}$. Given the moment generating functions of the two independent shocks in the pricing kernel ($\widetilde{\omega}_{c,t+1}$, $\widetilde{\omega}_{n,t+1}$), the risk free rate has an exact closed-form solution:

$$rf_t = -\ln \beta + \gamma \overline{c} + \gamma(1 - \phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1 + \lambda_t)^2 \sigma_c^2 - \left[\gamma(1 + \lambda_t)\sigma_n - \ln\left(1 + \gamma(1 + \lambda_t)\sigma_n\right)\right] n_t. \tag{15}$$

As in the CC model, the intertemporal substitution effect through s_t and the precautionary savings effect through λ_t counteract in determining the time variation in the risk free rate. The literature has proposed various modeling choices for the sensitivity function.[14] The present model proposes a strictly procyclical real rate, to be consistent with the few available empirical findings such as Ang, Bekaert, and Wei (2008). Specifically, λ_t is chosen such that the second-order Taylor approximation of the risk free rate is a constant as in the CC model:

$$\lambda_t = \begin{cases} \frac{1}{\overline{S}_t}\sqrt{1-2(s_t-\overline{s}_t)}-1, & s_t \leq s_{max,t} \\ 0, & s_t > s_{max,t} \end{cases} \tag{16}$$

where $\overline{s}_t = \log(\overline{S}_t)$ and $s_{max,t}$ are derived as functions of the free parameters and n_t:

$$\overline{S}_t = \sqrt{(\sigma_c^2 + \sigma_n^2 n_t)\frac{\gamma}{1-\phi_s}}, \tag{17}$$

$$s_{max,t} = \overline{s}_t + \frac{1}{2}(1-\overline{S}_t^2). \tag{18}$$

$\overline{S}_t$ and $s_{max,t}$ are time-varying equivalent to those in the CC model. As a result, the dynamics of the sensitivity function are determined by s_t and n_t. As in the CC model, when the consumption level is closer to the habit level (i.e., when s_t decreases), the sensitivity function increases. On the other hand, the uncertainty state variable n_t has a negative effect on λ_t through $\frac{1}{\overline{S}_t}$ and a positive effect through $\overline{s}_t$.

With this sensitivity function, the precautionary savings channel in the risk free rate contains a higher-order moment, which is different from the CC model. For instance, a third-order Taylor approximation of the risk free rate is given by:

$$rf_t \approx \underbrace{-\ln\beta + \gamma\overline{c} - \frac{(1-\phi_s)\gamma}{2}}_{\equiv rf_{CC}} + \frac{1}{3}\gamma^3(1+\lambda_t)^3 \underbrace{\sigma_n^3}_{<0} n_t. \tag{19}$$

rf_{CC} denotes the constant risk free rate as in the CC model (which assumes only Gaussian shocks). The appended precautionary savings term, $\frac{1}{3}\gamma^3(1+\lambda_t)^3\sigma_n^3 n_t$, is strictly procyclical given $\sigma_n < 0$. It captures that, in an extremely bad economic environment, the desire to save might eventually dominate the intertemporal substitution effect, resulting in a lower risk

[14]For instance, in the CC model, the two effects completely offset each other, resulting in a constant risk free rate. Wachter (2005, 2006) allow the intertemporal substitution effect to dominate in order to generate an upward sloping real yield curve, thus resulting in a countercyclical short rate. Bekaert and Engstrom (2017) propose a time-varying risk free rate such that the relative importance of the two effects varies over time, depending on the magnitudes of "good" and "bad" uncertainty state variables defined in their model.

free rate. Appendix Appendix C provides the derivations and graphical illustrations of the sensitivity function.

4.2 Approximate analytical solution

This model features three state variables: the procyclical log surplus consumption ratio (s_t), the countercyclical macroeconomic uncertainty (n_t), and the procyclical dividend-consumption comovement (b_t).[15] The model does not have an exact closed-form solution. In this section, I explore economic intuitions based on an approximate analytical solution.

4.2.1 Equity prices

I conjecture an approximate process for the log valuation ratio $pd_t \equiv \ln\left(\frac{P_t}{D_t}\right)$:

$$pd_t = A_0 + A_1 s_t + A_2 b_t + A_3 b_t^2 + A_4 n_t. \tag{20}$$

Then, I apply the Campbell–Shiller linearization to the log market return, $r^m_{t+1} = \ln\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right) \approx \Delta d_{t+1} + a_1 pd_{t+1} - pd_t + a_0$, where a_0 and a_1 are linearization constants. Given the shock assumptions and the pd conjecture, there are three types of shocks in this approximate log market return: Gaussian shocks, $\chi^2(1)$ shocks, and gamma shocks. Third, I apply a quadratic Taylor approximation to the Euler equation; $E_t\left[\exp(m_{t+1} + r^m_{t+1})\right]$ can be approximated by $\exp\left[E_t(m_{t+1} + r^m_{t+1}) + \frac{1}{2}V_t(m_{t+1} + r^m_{t+1})\right]$, according to Appendix Appendix D. The coefficients in the conjectured log valuation ratio are solved in closed form by equating the terms of the state variables; see Appendix Appendix E for the technical details and proofs.

I focus on the asset pricing implications of the new state variable introduced in this paper, procyclical dividend-consumption comovement, denoted by b_t. Through a pure cash flow (CF) effect, the valuation ratio can be interpreted as reflecting the outlook on future dividend growth, given that the expected value of the exponential of dividend growth increases with both the expected growth and the conditional variance.[16] In this model, the persistent procyclical

[15] The cyclicality of each state variable can be easily proved. Log surplus consumption ratio is procyclical because $Cov_t(s_{t+1}, \Delta c_{t+1}) = \lambda_t(\sigma_c^2 + \sigma_n^2 n_t) > 0$. As discussed in Section 3, macroeconomic uncertainty is countercyclical because $Cov_t(n_{t+1}, \Delta c_{t+1}) = \sigma_n \sigma_{nn} n_t < 0$, and dividend-consumption comovement is procyclical because $Cov_t(b_{t+1}, \Delta c_{t+1}) = \lambda_b \sigma_c^2 > 0$.

[16] The quadratic Taylor approximation shows $E_t\left[\exp(\Delta d_{t+1})\right] \approx \exp\left[E_t(\Delta d_{t+1}) + \frac{1}{2}V_t(\Delta d_{t+1})\right]$, where $E_t(\Delta d_{t+1})$ is $\bar{d} + \phi_d(V_{c,t} - \overline{V}_c)$ and $\frac{1}{2}V_t(\Delta d_{t+1})$ is $\frac{1}{2}(b_t^2\sigma_c^2 + V_d\sigma_d^2)$; notations and variables in this expression are introduced in Section 3. Importantly, $b_t^2\sigma_c^2$ is driven by the procyclical exposure of dividend growth to

dividend-consumption comovement entails a persistent procyclical dividend growth variance, $b_t^2\sigma_c^2 + V_d\sigma_d^2$. This variance becomes capitalized in stock prices. Therefore, this pure CF effect suggests a positive relationship between b_t^2 and pd_t.

In addition, there is a risk premium effect. The total risk premium to compensate changes in dividend growth can be approximated with $-Cov_t(m_{t+1}, \Delta d_{t+1}) = \gamma(1+\lambda_t)b_t\sigma_c^2$. This compensation increases with both the price-of-risk variable λ_t and the dividend-consumption comovement variable b_t. When a positive fundamental shock arrives, b_t and s_t increase and λ_t decreases simultaneously. If λ_t did not dominate, the model would generate a higher risk premium and a lower asset price during good times, which is counterintuitive.[17] However, the strongly countercyclical price of risk in this habit formation model is able to dominate so that there is an overall positive pd_t–b_t relationship through risk compensations.

Proposition 1. *Given appropriate parameter choices, there exist positive comovement effects on the equity valuation ratio,* $A_2, A_3 > 0$.

As for the other two state variables, the surplus consumption ratio effect as in CC implies a positive A_1. Unlike CC, my model implies competing effects of macroeconomic uncertainty that determine A_4. The CF effect of uncertainty is well understood: when macroeconomic uncertainty increases, future dividend growth is expected to decrease, driving down the current price. However, higher uncertainty also induces more precautionary savings, driving down the interest rate and lowering the total return demanded. This discount rate (DR) effect of uncertainty also appears in Bekaert and Engstrom (2017). Given appropriate parameter choices, the CF effect dominates the DR effect when extreme event shocks occur, yielding a negative A_4; in other times, A_4 becomes positive.

consumption growth, and $V_d\sigma_d^2$ is driven by the dividend-specific shock.

[17] This supports the second reason why a habit formation paradigm is more suitable to structurally examine the asset pricing implications of a procyclical source of risk; see the first paragraph of Section 4.

■ *Fact Check (f) and (g): Given the dividend growth dynamics in the new DGP and the valuation ratio conjecture, the conditional variances of the log valuation ratio and the log market return have the following approximate expressions:*

$$\begin{aligned} Var_t(pd_{t+1}) &\approx \varsigma_{pd} + \varsigma_1\lambda_t + \varsigma_2 b_t + \varsigma_3 n_t + \varsigma_4\lambda_t^2 + \varsigma_5 b_t^2 + \varsigma_6\lambda_t b_t + \varsigma_7\lambda_t n_t + \varsigma_8\lambda_t^2 n_t, \\ Var_t(r_{t+1}^m) &\approx \varsigma_{rm} + a_1^2\varsigma_1\lambda_t + \left[a_1^2\varsigma_2 + 2a_1\lambda_b\sigma_c^2\left(A_2 + 2A_3(1-\phi_b)\bar{b}\right)\right]b_t + a_1^2\varsigma_3 n_t + a_1^2\varsigma_4\lambda_t^2 \\ &+ \left(a_1^2\varsigma_5 + 2a_1\varsigma_2 + \sigma_c^2\right)b_t^2 + \left(a_1^2\varsigma_6 + 2a_1\varsigma_1\right)\lambda_t b_t + a_1^2\varsigma_7\lambda_t n_t + a_1^2\varsigma_8\lambda_t^2 n_t, \end{aligned}$$

where ς_{pd}, ς_{rm}, ς_1, ς_2, ς_3, ς_4, ς_5, ς_6, ς_8, a_1, λ_b, σ_c, ϕ_b, and $\bar{b}$ are strictly positive constants, and $\varsigma_7 = 2A_1A_4\sigma_n\sigma_{nn}$ is positive when the cash flow effect of n_t dominates the discount rate effect, and negative vice versa. The model has the potential to generate countercyclical variances if the countercyclical terms dominate.

4.2.2 The total amount of consumption risk

The approximate analytical solution of the total amount of risk is as follows:

$$\begin{aligned} &\underbrace{b_t\sigma_c^2}_{\text{[1]. Immediate cash flow covariance: dividend risk}} \\ +\ &\underbrace{a_1A_1\lambda_t\sigma_c^2}_{\text{[2]. Valuation covariance: benchmark time-varying amount of risk as in CC}} \\ +\ &\underbrace{\left[a_1A_2\lambda_b + 2a_1A_3(1-\phi_b)\bar{b}\lambda_b + 2a_1A_3\phi_b\lambda_b b_t\right]\sigma_c^2}_{\text{[3]. Valuation covariance: comovement risk}} \\ +\ &\underbrace{a_1\left[A_1\lambda_t\sigma_n^2 + A_4\sigma_{nn}\sigma_n\right]n_t}_{\text{[4]. Valuation covariance: downside risk and uncertainty risk,}}, \end{aligned} \tag{21}$$

where parameters $\sigma_c, \lambda_b, \phi_b, \bar{b}$, and σ_{nn} are positive and σ_n is negative, according to the DGP estimation results in Table 5; a_1 is a return linearization constant and is positive.

Term [1] captures the procyclical immediate cash flow covariance $Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$, or the amount of dividend risk. Meanwhile, the other three terms constitute the valuation covariance component of the total amount of consumption risk. Specifically, Term [2] captures the amount of risk implied from linearizing the original CC model (with a Gaussian consumption shock and constant volatility σ_c) and is strictly countercyclical. Term [3] captures the procyclical amount of dividend comovement risk through the valuation channel, which is new to the literature. Term [4] captures the amount of risk that is associated with the countercyclical macroeconomic uncertainty. The cyclicality of Term [4] is state-dependent. At a higher risk aversion state, the coefficient of macroeconomic uncertainty "$\left[A_1\lambda_t\sigma_n^2 + A_4\sigma_{nn}\sigma_n\right]$" is more likely to be positive, rendering Term [4] countercyclical. Derivations can be found in Appendix Appendix E.

In summary, the new procyclical dividend-consumption comovement state variable entails two new procyclical terms in the amount of risk: dividend risk (via cash flows) and comovement risk (via valuation). This analytical solution thus demonstrates the potential of my model to generate a realistic amount of risk in a habit formation framework.

■ *Fact Check (h): The model has the potential to generate a countercyclical valuation covariance, Term [2]+Term [3]+Term [4], if the procyclical terms are counteracted by the countercyclical terms.*

4.2.3 The equity premium

The equity premium in this approximate analytical solution is expressed as the product of a countercyclical price of risk $\gamma(1+\lambda_t)$ — which is consistent with CC — and a time-varying amount of risk that now comprises both procyclical and countercyclical terms according to Eq. (21) — which is new to the literature. These cyclical price-of-risk and amount-of-risk terms exhibit countervailing effects on the magnitude of the unconditional equity premium and the dynamic behavior of the conditional equity premium.

On the one hand, the introduction of the countercyclical uncertainty state variable n_t makes the asset riskier, since both the time-varying uncertainty and price of risk are higher during economic turmoil. From this perspective, a higher unconditional equity premium is expected. On the other hand, the introduction of the procyclical comovement state variable b_t lowers the level of the unconditional equity premium. This is because the amount of risk now contains procyclical terms that counteract the countercyclical amount-of-risk terms and the countercyclical price of risk. As a result, the asset becomes less risky.

Moreover, in contrast to the CC model, the conditional equity premium no longer monotonically increases when consumption drops. This is because there are two types of consumption shocks in the economy. A negative *event* shock increases both the price of risk and the amount of risk, resulting in a lower asset price and a higher equity risk premium; this effect is consistent with conventional theories. As the core theoretical contribution of this paper, a negative *fundamental* shock increases the price of risk while lowering the amounts of dividend risk and comovement risk, resulting in competing effects on the conditional equity premium. This new fundamental channel cannot be neglected because, empirically, this fundamental shock accounts for more than 80% of the total consumption variance in a long sample (see discussions in Section 3.2). Therefore, the ultimate impact of a current consumption shock realization on the conditional equity premium can be nonlinear.

4.3 Numerical solution

To identify the implications of each state variable, I conduct an overlaying numerical analysis. The baseline model, referred to as M(1) in the rest of the paper, is an adapted Campbell and Cochrane (1999) model that features homoskedastic fundamentals and constant dividend-consumption comovement; the time-varying surplus consumption ratio is the only state variable. Then, M(2), building on M(1), is an adapted Bekaert and Engstrom (2017) model that incorporates countercyclical macroeconomic uncertainty as the second state variable. Finally, my model, labeled as M(3), overlays M(2) with procyclical dividend-consumption comovement in the dividend growth process. Appendix Appendix C provides mathematical descriptions of M(1) and M(2). All three models price dividend claims.

Section 4.3.1 describes the calibration of the non-DGP parameters. Then, I evaluate the fit of the eight cyclical moments inspired by the Duffee Puzzle in Section 4.3.2 and the fit of conventional unconditional asset moments in Section 4.3.3.

4.3.1 Calibration and simulation

Table 7 summarizes the four non-DGP parameters. The utility curvature parameter γ is fixed at 2. As is commonly assumed in the literature, the persistence coefficient of the s_t process, ϕ_s, equals the AR(1) coefficient of monthly log valuation ratio. The benchmark constant risk free rate, rf_{CC}, as it appears in Eq. (19), is chosen to match the average monthly real short rate (proxied by the difference between the change in the log nominal 90-day Treasury index constructed by CRSP and the continuously compounded inflation rate). β is the time discount parameter inferred from the rf_{CC} equation.

The log valuation ratios are solved numerically using the "series method" from Wachter (2005). M(1) is solved using a one-dimensional grid (20×1) for the one state variable: the log surplus consumption ratio. M(2) is solved over a two-dimensional grid (20×20) for the two state variables: the log surplus consumption ratio and macroeconomic uncertainty. The final model M(3) uses a three-dimensional grid (20×20×20) for all three state variables. Appendix Appendix F discusses the dependences of the valuation ratio on the three state variables in all three models that are consistent with the analytical predictions earlier. Then, for each model, I simulate the shocks for 100,000 months given their distributional assumptions and parameter estimates, and construct the state variable processes accordingly. Based on the grid solutions, I apply the piecewise polynomial cubic interpolation for M(1), and the piecewise polynomial spline interpolation for M(2) and M(3) to obtain the log valuation ratio for each simulated month given the state variable values. All the reported theoretical moments in this paper are calculated using the second half of the simulated dataset.

4.3.2 The eight cyclical moments

Given that this research focuses on cyclical behavior of conditional moments, one challenge is to identify realistic recessions in a simulated consumption-based economy. For this purpose, I develop an algorithm based on the simulated consumption growth such that the algorithm mimics the identification of NBER recessions that are based on patterns in GDP growth. Details and empirical tests are provided in Appendix Appendix G.

Table 8 examines the closeness between the empirical and simulation asset moments of Facts (a)–(h), using recession and non-recession subsamples. On fitting Facts (a)–(e), most simulation moment point estimates of both recession and non-recession periods in M(3) are within 95% confidence intervals of the actual data point estimates. The only exception is the recession-period dividend growth volatility, according to row "$\sigma(\Delta d)$ ($I_{rece.} = 1$)": due to the non-Gaussian nature of the simulated macroeconomic uncertainty n_t, extreme values are more likely to appear during recessions, which results in an extremely volatile conditional mean of dividend growth. However, the *conditional* dividend growth variance is strictly procyclical, according to row "$\sigma_t(\Delta d_{t+1}) \sim I_{rece.,t}$" in Table 8. By design, M(3) outperforms M(1) and M(2) on matching the joint dynamics of dividend growth and consumption growth. While M(1) and M(2) generate recession and non-recession simulation moments that cannot be rejected by the empirical point estimates, both models fail to fit any of the cyclicality tests of conditional moments in Facts (a)–(e).

On fitting Facts (f) and (g), M(1) — an adapted CC model — generates recession and non-recession volatilities of $r^m - \Delta d$ and r^m that are significantly lower than the empirical point estimates. Meanwhile, M(2) and M(3), which allow for countercyclical consumption growth uncertainty n_t, improve the fit and imply more realistic magnitude. It is noteworthy that the new comovement state variable b_t in M(3) dampens the volatilities of $r^m - \Delta d$ and r^m. As suggested by the analytical solution in Eq. (21), the procyclical dividend risk and comovement risk counteract the countercyclical terms in the amount of risk. As a result, assets in M(3) are less risky: when consumption drops during recessions, asset prices do not drop as much as those in M(2).

On fitting Fact (h), M(1) and M(2) tend to generate non-recession valuation covariances that are unrealistically high and rejected by data, e.g., 4.0956×10^{-5} in M(1) and 3.1658×10^{-5} in M(2) versus 1.7436×10^{-5} in data. The reason why M(1) generates a high valuation covariance despite the low price variation is that there is a high correlation (around 0.7 in my simulation) between the price dynamics and the consumption growth innovations through the surplus consumption ratio. This correlation decreases to around 0.25 in M(2) due to the introduction of a second state variable to the price dynamics. Despite the misfit of M(2) in matching the magnitudes, allowing for countercyclical macroeconomic uncertainty clearly generates more countercyclicality in the valuation covariance. The difference between recession and non-recession valuation covariances is wider in M(2) than in M(1). One major improvement of M(3) over M(2) is that both the recession and the non-recession valuation covariances are now lower and statistically closer to the data point estimates. This is largely because of the lower price variability shown in row (f). Therefore, M(3) fits Fact (h) in terms of both cyclicality and magnitude.

This overlaying numerical analysis so far demonstrates that price dynamics become different after the introduction of the new procyclical comovement state variable in M(3). Finally, as shown in the last two rows in Table 8, M(3) is the only model (of the three) that simulates point estimates of the recession and non-recession amount of consumption risk that cannot be rejected by data. In contrast, M(1) and M(2) generate amount of consumption risk point estimates that are significantly higher than the empirical point estimates. As a result, M(3) potentially addresses the Duffee Puzzle by generating a more realistic magnitude of consumption risk.[18]

To provide over-identification tests, Table 9 evaluates the fit of the three models in terms of 17 unconditional moments including the eight puzzle moments: $\sigma(\Delta c)$, $\sigma(\Delta d)$, $Corr(\Delta d, \Delta c)$, $Cov(\Delta d, \Delta c)$, $\beta(\Delta d, \Delta c)$, $\sigma(r^m - \Delta d)$, $\sigma(r^m)$, and $Cov(r^m - \Delta d, \Delta c)$. As shown in the first 13 rows, despite the different DGP assumptions among the three overlaying models, all their unconditional simulation moments are shown to match the data well. This result in turn supports the argument that comparing unconditional moments across various conditional models is not an informative test and should not be used as the primary test. From the last four rows, M(1) generates significantly smaller unconditional volatilities of $r^m - \Delta d$ and r^m than the empirical counterparts, which is expected because one of the main concerns about the original CC model is the small *pd* variability. Given the single-state variable economy, M(1) also generates unrealistically high unconditional $Cov(r^m - \Delta d, \Delta c)$ and $Cov(r^m, \Delta c)$ than data. On the other hand, M(2) and M(3) are not rejected by data in terms of these four unconditional moments. The magnitude of the unconditional return-consumption covariance in M(3), 3.1738×10^{-5}, is the closest to its data counterpart, 2.4822×10^{-5}.

4.3.3 Conventional moments

Finally, Table 10 reports the fit of the models with respect to a set of conventional unconditional moments. Incorporating procyclical dividend-consumption comovement, M(3) implies a slightly lower unconditional equity premium, 5.6524%, than M(2), 6.2414%. This is consistent with the economic intuition mentioned in Section 4.2.3: The amount of consumption risk in M(3) now contains procyclical immediate cash flow risk and comovement risk, resulting in a

[18]Note that the non-recession point estimate of return-consumption covariance using data (1.8546×10^{-5}) is smaller in magnitude than the recession estimate (3.3894×10^{-5}), although both are statistically indifferent according to the t test. The slightly higher recession-sample estimate is expected because the conditional means of both return and consumption growth series are expected to comove more during recessions. Ideally, one can calculate the $Cov\left[r^m_{t+1} - E_t(r^m_{t+1}), \Delta c_{t+1} - E_t(\Delta c_{t+1})\right]$ to solve this issue; however, numerical solution of such habit formation models does not generate $E_t(r^m_{t+1})$.

less risky asset. In addition, consistent with the literature, the introduction of countercyclical macroeconomic uncertainty increases the level of equity premium significantly, from 3.8751% in M(1) to 6.2414% in M(2). The uncertainty state variable overall introduces additional countercyclical dynamics into the amount of risk, resulting in a riskier asset and a lower average valuation ratio (as also shown in Table 10).

Because the procyclical comovement state variable contributes positively to the valuation ratio, M(3) implies a valuation ratio volatility that is the highest and closest to the data point estimate among the three models. The market return volatility implied by M(3), 14.7234%, is slightly smaller than that implied by M(2), which can be explained analytically as follows. Unconditional market return variance, given return linearization, can be roughly decomposed into three components: (a) unconditional variance of $a_1 pd_{t+1} - pd_t$, (b) unconditional variance of Δd_{t+1}, and (c) unconditional covariance between $a_1 pd_{t+1} - pd_t$ and Δd_{t+1}. In particular, given the law of iterated expectations, $Cov(a_1 pd_{t+1} - pd_t, \Delta d_{t+1}) = E\left[Cov_t(pd_{t+1}, \Delta d_{t+1})\right]$ contains term $E(A_1 \lambda_t \bar{b} \sigma_c^2)$ in M(1) and M(2) but term $E(A_1 \lambda_t b_t \sigma_c^2)$ in M(3), and $E(A_1 \lambda_t b_t \sigma_c^2)$ is smaller than $E(A_1 \lambda_t \bar{b} \sigma_c^2)$ because of the negative correlation between λ_t and b_t. This negative correlation reflects the nonlinear effect of a fundamental consumption shock in asset prices, through procyclical comovement risk and through countercyclical risk aversion simultaneously. Section 4.2.3 discusses this unique implication of my model.

The implied Sharpe Ratio from M(3), 0.3888, is the closest to the data point estimate because of the smaller implied equity premium. Moreover, the kurtosis moment is matched statistically well by all three models. M(2) and M(3) generate the same risk free rate dynamics as they differ only in the cash flow part; this average risk free rate is statistically close to the data counterpart.

5 Conclusion

Inspired by the Duffee Puzzle, this paper aims to understand the procyclicality of the conditional covariance between market returns and consumption growth, and to potentially accommodate this procyclical amount of risk in a consumption-based asset pricing model. To achieve these aims, I first show empirically that the conditional covariance between the immediate cash flow part of market returns (dividend growth) and consumption growth is (1) procyclical and (2) a consistent source of procyclicality in the return-consumption covariance. This is the core empirical finding of the paper. Then, I devise a new DGP that is able to simultaneously accommodate procyclical dividend-consumption comovement and countercyclical consumption growth volatility. Finally, I solve a variant of the Campbell and Cochrane model incorporating this new DGP. The approximate analytical solution suggests that the procyclical dividend-consumption comovement, as a new state variable, entails two new procyclical terms in the amount of consumption risk: dividend risk (via cash flows) and comovement risk (via valuation). These procyclical terms, according to the numerical solution, play an important role in generating a realistic magnitude of the total amount of consumption risk in a habit formation model. Moreover, in contrast to the Campbell-Cochrane model, the conditional equity premium no longer increases monotonically when a negative consumption shock arrives because it might lower the dividend risk and the comovement risk while increasing the price of risk.

Appendix A Discussions on alternative DGPs

In this appendix section, I discuss why it might be difficult to construct an alternative DGP of dividend growth and consumption growth that jointly satisfies Facts (a)–(e). The key challenge is to imply both (1) procyclical dividend growth variance and dividend-consumption comovement and (2) countercyclical consumption growth variance. A one-consumption shock assumption is unattractive because the model need an extremely procyclical exposure of dividend growth to the heteroskedastic consumption shock, which is impossible to achieve without certain assumptions on the parameters. In Section 3, my DGP assumes two types of consumption shocks: the fundamental shock enters the dividend growth process with a procyclical exposure, while the event shock determines the heteroskedasticity of consumption growth. The model-implied conditional moments have the potential to jointly satisfy the five stylized facts, as confirmed by Table 5 of Section 3.

Alternatively, one could assume a "constant" exposure of dividend growth to the fundamental consumption shock — which is commonly assumed in the literature — and a "procyclical" fundamental shock conditional variance. This way, during each period, the consumption growth disturbance is driven by a heteroskedastic Gaussian shock with procyclical volatility and a heteroskedastic gamma shock with countercyclical volatility. With proper parameter values, this model can generate procyclical dividend-consumption comovement and countercyclical consumption variance. However, this alternative DGP has two potential problems. The first problem is that the identification of consumption growth variance is likely to be difficult. The analytical expression of the consumption growth conditional variance is now the sum of a procyclical component from the fundamental shock and a countercyclical component from the event shock. Given that a Gaussian distribution is symmetric and not bounded, a heteroskedastic Gaussian fundamental shock might act as the event shock trying to fit the left-tail events in the estimation. This likely results in countercyclical volatility of fundamental shock as well. Granted, one can restrict the fundamental shock volatility to be procyclical by restricting signs of certain parameters; however, it is difficult to interpret results of a constrained estimation. The second problem is that other empirical facts might be easily violated. For instance, while the data show significant and negative correlation (-0.2090) between the conditional variances of dividend growth and consumption growth, this alternative DGP will generate a strictly positive correlation. This occurs because the conditional variances of consumption growth and dividend growth now have constant and positive exposures to the conditional variance of the fundamental consumption shock, respectively. Given these two potential problems (i.e., estimation difficulties and violation of other empirical facts), this alternative DGP is not suitable.

Appendix B Estimation procedure for the new DGP in Section 3

Given that there is no feedback from the dividend growth process to the consumption growth process, I conduct a two-step estimation procedure. The first step estimates the consumption growth system. I use a filtration-based maximum likelihood methodology in Bates (2006) to estimate the latent macroeconomic uncertainty state variable n_t and the two consumption shocks, the fundamental shock $\widehat{\widetilde{\omega}}_{c,t+1}$ and the event shock $\widehat{\widetilde{\omega}}_{n,t+1}$, where "$\widehat{\ }$" indicates the estimated variables. The Bates method is particularly suitable to filter non-Gaussian shocks. Then, the consumption growth conditional variance and its long-run average are then obtained, $\widehat{V}_{c,t}$ and $\widehat{\overline{V}}_c$ respectively. The second step takes the dividend growth data Δd_{t+1} and state variable and shock estimates from the first step $\{\widehat{V}_{c,t}, \widehat{\overline{V}}_c, \widehat{\sigma}_c, \widehat{\widetilde{\omega}}_{c,t+1}\}$. To provide estimation convenience, dividend growth is first projected onto $\widehat{V}_{c,t} - \widehat{\overline{V}}_c$ to obtain the estimates for $\{\overline{d}, \phi_d\}$. The rest of the dividend growth system is then estimated by maximizing the sum of the log likelihoods of the implied cash flow-specific shock $\widetilde{\omega}_{d,t+1}$, which follows a centered gamma density function. The MLE estimation does not impose constraints on the non-negativity of b_t estimates, but imposes one constraint to ensure a valid gamma density function for $\widetilde{\omega}_{d,t+1}$ at any time t; otherwise, a gamma density cannot be defined. If $\widetilde{\omega}_{d,t+1}$ in the data is more right-tailed and is hence bounded below, then σ_d will be estimated to be positive and the constraint is as follows: $-\sigma_d V_d \leq \min_{\forall t \in 1,\ldots,T} \left(\Delta d_{t+1} - \widehat{\overline{d}} - \widehat{\phi}_d(\widehat{V}_{c,t} - \widehat{\overline{V}}_c) - b_t \widehat{\widetilde{\omega}}_{c,t+1}\right)$. On the other hand, if $\widetilde{\omega}_{d,t+1}$ is more left-tailed and is hence bounded above, then σ_d will be estimated to be negative with the following constraint: $-\sigma_d V_d \geq \max_{\forall t \in 1,\ldots,T} \left(\Delta d_{t+1} - \widehat{\overline{d}} - \widehat{\phi}_d(\widehat{V}_{c,t} - \widehat{\overline{V}}_c) - b_t \widehat{\widetilde{\omega}}_{c,t+1}\right)$.

Appendix C Intermediate and final models in Section 4

In this appendix section, I provide details of the two intermediate models in the overlaying framework and compares them with the final model of the paper, referred to as M(3). M(1) is an adapted CC model with constant macroeconomic uncertainty and constant dividend-consumption comovement, while M(2) builds on M(1) and allows for countercyclical macroeconomic uncertainty. The final model, M(3), allows for both countercyclical macroeconomic uncertainty and procyclical dividend-consumption comovement. The DGPs of the fundamentals are as follows:

$$\begin{aligned}
\text{M(1):}\ & \Delta c_{t+1} = \overline{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, && \text{(C.1)}\\
& \widetilde{\omega}_{c,t+1} \sim N(0,1), \widetilde{\omega}_{n,t+1} \sim \Gamma(\overline{n},1) - \overline{n}, \\
& \Delta d_{t+1} = \overline{d} + \overline{b}\sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, && \text{(C.2)}\\
& \widetilde{\omega}_{d,t+1} \sim \Gamma(V_d,1) - V_d, \\
& \overline{c} = 0.0025, \sigma_c = 0.0029, \sigma_n = -0.0023, \overline{n} = 0.3742, \\
& \overline{d} = 0.0015, \sigma_d = 0.000123, V_d = 8933.5172, \overline{b} = 0.0944; \\
\text{M(2):}\ & \Delta c_{t+1} = \overline{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, && \text{(C.3)}\\
& \widetilde{\omega}_{c,t+1} \sim N(0,1), \widetilde{\omega}_{n,t+1} \sim \Gamma(n_t,1) - n_t, \\
& n_{t+1} = (1-\phi_n)\overline{n} + \phi_n n_t + \sigma_{nn}\widetilde{\omega}_{n,t+1}, && \text{(C.4)}\\
& V_{c,t} = Var_t(\Delta c_{t+1}) = \sigma_c^2 + \sigma_n^2 n_t, && \text{(C.5)}\\
& \overline{V}_c = E(V_{c,t}), && \text{(C.6)}\\
& \Delta d_{t+1} = \overline{d} + \phi_d (V_{c,t} - \overline{V}_c) + \overline{b}\sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, && \text{(C.7)}\\
& \widetilde{\omega}_{d,t+1} \sim \Gamma(V_d,1) - V_d, \\
& \overline{c} = 0.0025, \sigma_c = 0.0029, \sigma_n = -0.0023, \overline{n} = 0.3742, \phi_n = 0.9500, \sigma_{nn} = 0.2772, \\
& \overline{d} = 0.0015, \phi_d = -630.8768, \sigma_d = 0.000123, V_d = 8933.5172, \overline{b} = 0.0944; \\
\text{M(3):}\ & \text{The new DGP in this paper (as shown in Table 5).}
\end{aligned}$$

The log surplus consumption ratios in all three models follow an AR(1) process with time-varying sensitivities to the consumption growth innovation. The sensitivity functions are chosen as follows:

$$\lambda_t = \begin{cases} \frac{1}{\overline{S}_t}\sqrt{1-2(s_t-\overline{s}_t)} - 1, & s_t \le s_{max,t} \\ 0, & s_t > s_{max,t} \end{cases}, \tag{C.8}$$

$$\overline{s}_t = \log(\overline{S}_t), \tag{C.9}$$

$$s_{max,t} = \overline{s}_t + \frac{1}{2}(1-\overline{S}_t^2), \tag{C.10}$$

$$\text{M(1):}\ \overline{S}_t = \sqrt{(\sigma_c^2 + \sigma_n^2 \overline{n})\frac{\gamma}{1-\phi_s}}, \tag{C.11}$$

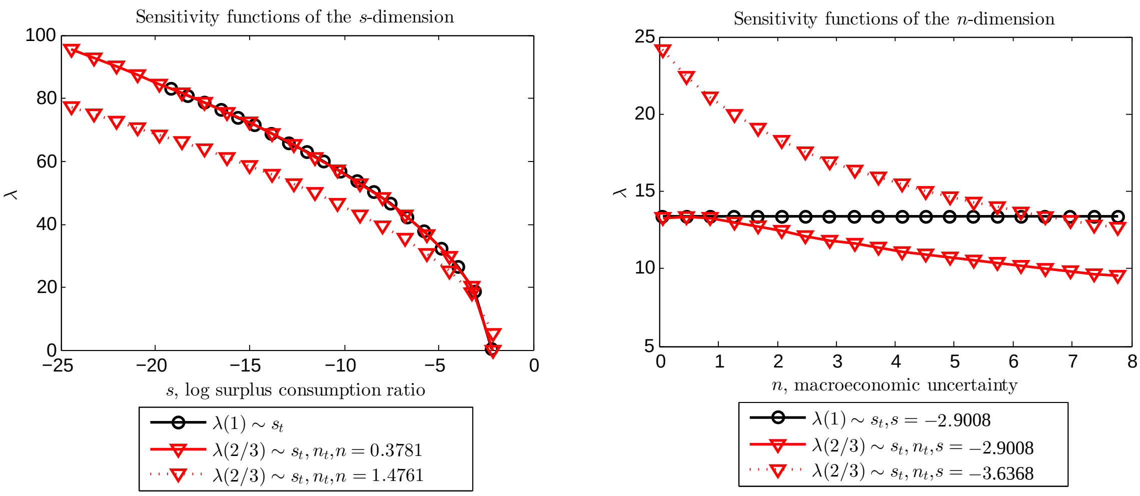
$$\text{M(2/3):}\ \overline{S}_t = \sqrt{(\sigma_c^2 + \sigma_n^2 n_t)\frac{\gamma}{1-\phi_s}}. \tag{C.12}$$

Given the sensitivity functions, the real risk free rates are time-varying with a higher moment appended to reflect the non-Gaussian nature of the consumption event shock:

$$\begin{aligned}
\text{M(1):}\ rf_t &= -\ln\beta + \gamma\overline{c} + \gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \left[\gamma(1+\lambda_t)\sigma_n - \ln\left(1+\gamma(1+\lambda_t)\sigma_n\right)\right]\overline{n} \\
&\approx -\ln\beta + \gamma\overline{c} + \underbrace{\gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_n^2\overline{n}}_{\text{fix}=-\frac{(1-\phi_s)\gamma}{2}} + \frac{1}{3}\gamma^3(1+\lambda_t)^3\sigma_n^3\overline{n}; \quad \text{(C.13)}
\end{aligned}$$

$$\begin{aligned}
\text{M(2/3):}\ rf_t &= -\ln\beta + \gamma\overline{c} + \gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \left[\gamma(1+\lambda_t)\sigma_n - \ln\left(1+\gamma(1+\lambda_t)\sigma_n\right)\right] n_t \\
&\approx -\ln\beta + \gamma\overline{c} + \underbrace{\gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_n^2 n_t}_{\text{fix}=-\frac{(1-\phi_s)\gamma}{2}} + \frac{1}{3}\gamma^3(1+\lambda_t)^3\sigma_n^3 n_t. \quad \text{(C.14)}
\end{aligned}$$

The calibration plots of the sensitivity functions can be illustrated by fixing one state variable at a time. The left plot depicts the relationship between λ_t and s_t, and the plot fixes at $E(n_t)$ (0.3781) and its 95th percentile (1.4761) given the simulated n_t path. The right plot depicts the relationship between λ_t and n_t, and the plot fixes at $E(s_t)$ (-2.9008) and its 5th percentile (-3.6368). The second option, the 95th percentile of n_t or 5th percentile of s_t, represents an extremely bad economic environment.



Appendix D Quadratic approximation of the moment generating function of a random variable that receives independent Gaussian, χ^2, and gamma shocks

Suppose a random variable x receives three independent shocks,

$$\begin{aligned}
&x = \mu + x_1\omega + x_2(\omega^2 - 1) + x_3(\varepsilon - \alpha),\\
&\omega \sim N(0,1),\\
&\omega^2 \sim \chi^2(1),\\
&\varepsilon \sim \Gamma(\alpha, 1),
\end{aligned} \tag{D.1}$$

where μ is the unconditional mean of variable x, and $\{x_1, x_2, x_3\}$ are constant coefficients. Recall the moment generating function (MGF) is $mgf_\omega(\nu) = \exp(\nu^2/2)$ for a standard Gaussian shock, $mgf_{\omega^2}(\nu) = (1-2\nu)^{-1/2}$ for a χ^2 shock, and $mgf_\varepsilon(\nu) = (1-\nu)^{-\alpha}$ for a gamma shock with a unit scale parameter and shape parameter equal to α. Therefore, the MGF of x, $mgf_x(\nu) = E[\exp(\nu x)]$, is as follows,

$$\begin{aligned}
mgf_x(\nu) &= \exp(\nu\mu) E_t[\exp(\nu x_1 \omega + \nu x_2(\omega^2 - 1) + \nu x_3(\varepsilon - \alpha))]\\
&= \exp(\nu\mu - \nu x_2 - \nu x_3 \alpha) mgf_\omega(\nu x_1) mgf_{\omega^2}(\nu x_2) mgf_\varepsilon(\nu x_3)\\
&= \exp(\nu\mu - \nu x_2 - \nu x_3\alpha) \exp\left\{\frac{1}{2}(\nu x_1)^2\right\} (1 - 2\nu x_2)^{-1/2} (1 - \nu x_3)^{-\alpha}\\
&= \exp(\nu\mu - \nu x_2 - \nu x_3\alpha) \exp\left\{\frac{1}{2}(\nu x_1)^2 - \frac{1}{2}\ln(1 - 2\nu x_2) - \alpha\ln(1 - \nu x_3)\right\}.
\end{aligned} \tag{D.2}$$

It can be easily shown that the quadratic approximation to $\ln(1-z)$ is $-z - \frac{1}{2}z^2$. The quadratic approximation to $mgf_x(\nu)$ yields:

$$\begin{aligned}
mgf_x(\nu) &\approx \exp(\nu\mu - \nu x_2 - \nu x_3\alpha) \exp\left\{\frac{1}{2}(\nu x_1)^2 + \nu x_2 + (\nu x_2)^2 + \nu x_3\alpha + \frac{1}{2}(\nu x_3)^2\alpha\right\}\\
&= \exp(\nu\mu) \exp\left\{\frac{1}{2}(\nu x_1)^2 + (\nu x_2)^2 + \frac{1}{2}(\nu x_3)^2\alpha\right\}\\
&= \exp(\nu E(x)) \exp\left\{\frac{1}{2}\nu^2 V(x)\right\}.
\end{aligned} \tag{D.3}$$

Define $X = \exp(x)$ and set $\nu = 1$,

$$E(X) \approx \exp\left\{E(x) + \frac{1}{2}V(x)\right\}. \tag{D.4}$$

Appendix E Approximate analytical solution of M(3)

In this appendix section, I solve the theoretical model in Section 4 with an approximate analytical solution. There are three approximations. The first approximation conjectures the log valuation ratio pd_t, $pd_t = A_0 + A_1 s_t + A_2 b_t + A_3 b_t^2 + A_4 n_t$. The second approximation applies the Campbell–Shiller linearization to the log market return, $r^m_{t+1} = \Delta d_{t+1} + a_1 pd_{t+1} - pd_t + a_0$. The log market return can be approximately expressed as a linear function of the state variables and four independent shocks to the economy:

$$
\begin{aligned}
r^m_{t+1} = {} & \overline{d} - \phi_d \sigma_d^2 \overline{n} + a_1 \left(A_0 + A_1(1-\phi_s)\overline{s} + A_2(1-\phi_b)\overline{b} + A_3((1-\phi_b)\overline{b})^2 + A_4(1-\phi_n)\overline{n} \right) - A_0 + a_0 \\
& + A_1(a_1\phi_s - 1)s_t + \left(a_1 A_2 \phi_b + 2a_1 A_3 (1-\phi_b)\overline{b}\phi_b - A_2 \right) b_t \\
& + A_3(a_1\phi_b^2 - 1)b_t^2 + \left(a_1 A_4 \phi_n - A_4 + \phi_d \sigma_d^2 \right) n_t \\
& + \left(a_1 A_1 \lambda_t + a_1 A_2 \lambda_b + 2a_1 A_3 (1-\phi_b)\overline{b}\lambda_b + (1 + 2a_1 A_3 \phi_b \lambda_b) b_t \right) \sigma_c \widetilde{\omega}_{c,t+1} \\
& + a_1 A_3 \lambda_b^2 \left(\sigma_c \widetilde{\omega}_{c,t+1} \right)^2 + a_1 \left(A_1 \lambda_t \sigma_n + A_4 \sigma_{nn} \right) \widetilde{\omega}_{n,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}. \qquad \text{(E.1)}
\end{aligned}
$$

The third approximation applies quadratic approximation to the MGF of random variable $m_{t+1} + r^m_{t+1}$ using the proof in Appendix Appendix D, given the composite shock structure. Finally, by equating the terms for the state variables, the coefficients in the valuation ratio equation are solved:

$$A_1 = \frac{\gamma(1-\phi_s)}{1 - a_1\phi_s} > 0, \qquad \text{(E.2)}$$

$$A_2 = \frac{(1 + 2a_1A_3\phi_b\lambda_b)\left[\gamma(1+\lambda_t) - (a_1A_1\lambda_t + 2a_1A_3(1-\phi_b)\overline{b}\lambda_b)\right]\sigma_c^2 - 2a_1A_3(1-\phi_b)\phi_b\overline{b}}{a_1\phi_b - 1 + a_1\lambda_b(1 + 2a_1A_3\phi_b\lambda_b)\sigma_c^2} > 0. \qquad \text{(E.3)}$$

$$A_3 = \frac{-2a_1\phi_b\lambda_b\sigma_c^2 + 1 - a_1\phi_b^2 \pm \sqrt{(2a_1\phi_b\lambda_b\sigma_c^2 - 1 + a_1\phi_b^2)^2 - 4a_1^2\phi_b^2\lambda_b^2\sigma_c^4}}{4a_1^2\phi_b^2\lambda_b^2\sigma_c^2} > 0. \qquad \text{(E.4)}$$

$$A_4 = \frac{\xi_t \pm \sqrt{\xi_t^2 - 2\sigma_{nn}^2 a_1^2 \left(\phi_d\sigma_d^2 + \frac{1}{2}\left(A_1\lambda_t a_1 - \gamma(1+\lambda_t) \right)^2 \sigma_n^2 \right)}}{\sigma_{nn}^2 a_1^2} > 0, \qquad \text{(E.5)}$$

$$\xi_t = 1 - \phi_n a_1 - a_1^2 A_1 \lambda_t \sigma_n \sigma_{nn} + \gamma(1+\lambda_t) a_1 \sigma_n \sigma_{nn}. \qquad \text{(E.6)}$$

An approximate analytical solution of equity premium is derived, given the quadratic approximation:

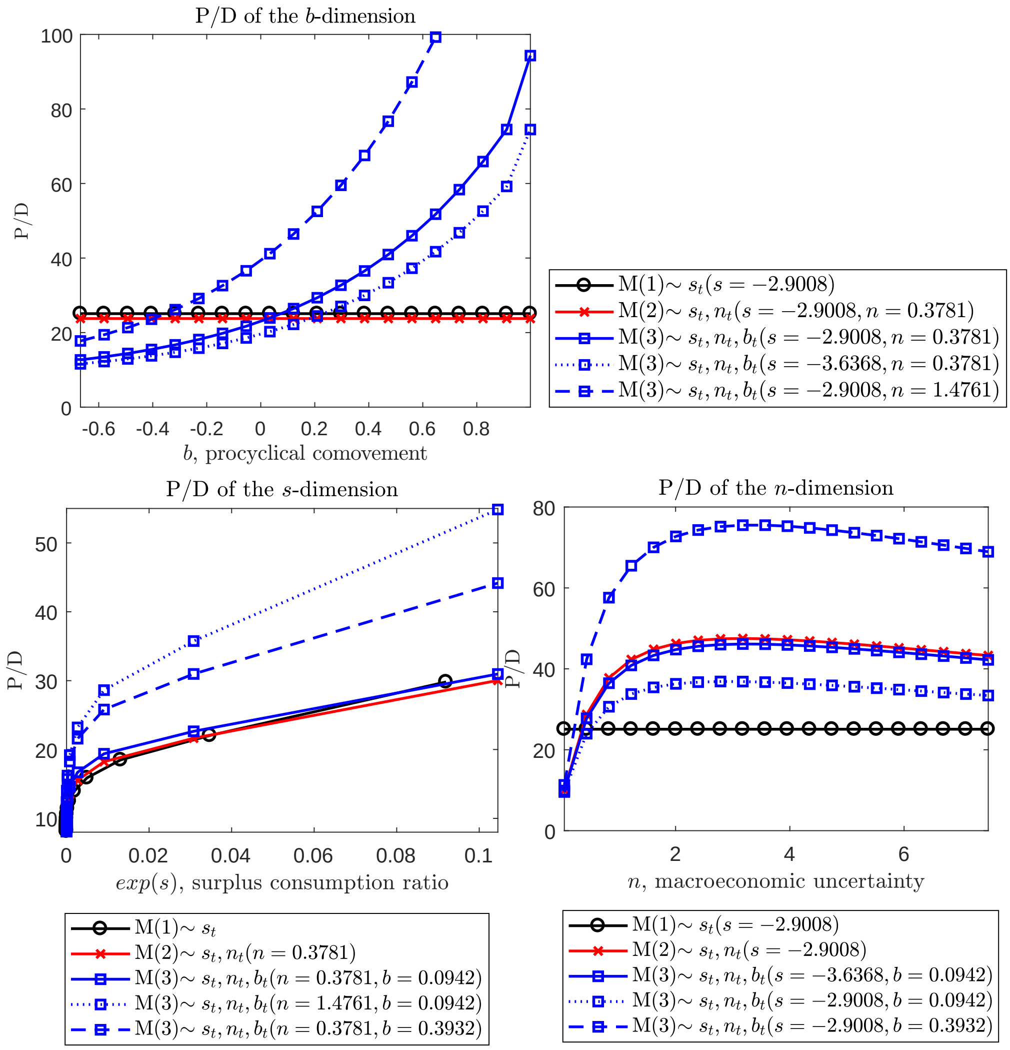
$$
\begin{aligned}
E_t\left(r^m_{t+1}\right) - rf_t + \frac{1}{2}Var_t\left(r^m_{t+1}\right) & \approx -Cov_t\left(r^m_{t+1}, m_{t+1}\right) \\
& = \gamma(1+\lambda_t) \times \{ a_1A_1\lambda_t\sigma_c^2 + \left[a_1A_2\lambda_b + 2a_1A_3(1-\phi_b)\overline{b}\lambda_b + (1 + 2a_1A_3\phi_b\lambda_b)b_t \right]\sigma_c^2 \\
& + a_1\left[A_1\lambda_t\sigma_n^2 + A_4\sigma_{nn}\sigma_n \right] n_t \}. \qquad \text{(E.7)}
\end{aligned}
$$

Appendix F Dependences of the valuation ratio on b, s, and n

The three plots below depict the dependences of the valuation ratio (P/D) on the three state variables, respectively, for all three models. M(1) is spanned by the log surplus consumption ratio state variable s only and is depicted in solid black lines with circles; M(2) is spanned by s and the macroeconomic uncertainty state variable n and is depicted in solid red lines with "x"; M(3) is spanned by s, n, and the comovement state variable b and is depicted in blue lines with squares. The dimension is reduced by fixing the other state variables at their mean values (mean of s, or $E(s)$: -2.9008; mean of n, or $E(n)$: 0.3781; mean of b, or $E(b)$: 0.0942) and critical values in a generally bad economic environment (e.g., 5th percentile in the simulated s path: -3.6368; 95th percentile in the simulated n path: 1.4761; 95th percentile in the simulated b path: 0.3932). Hence, the lines in the following three plots can be interpreted with a conditional statement.

In the first plot below, the positive association between the valuation ratio and the comovement state variable b confirms the analytical prediction. The M(1) and M(2) horizontal lines intersect the $E(s)$–$E(n)$ plane of M(3) at around $b = 0.094$, which is expected because $\widehat{\overline{b}} = 0.094$ according to Table 5. The convex increasing pattern indicates that the effect of b on asset prices is stronger when the level of dividend-consumption comovement is higher. In the present calibration, the mean (and in fact median) of the simulated b_t is around 0.1 and its 95th value is 0.39.

The valuation ratio implied by M(3) at the "lower 5th s"–"$E(n)$" plane ($s = -3.6368, n = 0.3781$) lays below the "$E(s)$"–"$E(n)$" plane ($s = -2.9008, n = 0.3781$), indicating a positive relationship between P/D and s; this relationship is confirmed by the second plot below. Similarly, the valuation ratio at the "$E(s)$"–"higher 95th n" plane ($s = -2.9008, n = 1.4761$) is above the $E(s)$–$E(n)$ plane, indicating a positive relationship between PD and n; this relationship is confirmed by the third plot below. It is noteworthy that the average and 95th percentile of the simulated n_t path, 0.3781 and 1.4761, respectively, are within the lower region in the third plot where the DR effect still dominates the traditional CF effect (see discussions in Section 4.2.1); thus, a positive relationship between the valuation ratio and n is expected. However, the hump shape is interesting as it captures that, under extremely high macroeconomic uncertainty (when n increases), the CF effect then dominates and stock prices decrease with uncertainty.



Appendix G Identifying recessions using simulated monthly consumption growth

In this appendix section, I develop an algorithm to identify realistic recessions based on the simulated consumption growth rates such that the algorithm mimics the NBER recession indicator, which is based on the quarterly GDP growth rates. Here are the steps:

1. *Quarterly Growth*: Aggregate the monthly consumption growth into a quarterly frequency.
2. *Standardization*: De-center the quarterly consumption growth by a long-term moving average (e.g., a 49-quarter moving average using 24 quarters before and after), and divide it with its long-term or unconditional standard deviation to obtain the standardized consumption growth rates.
3. *Fundamental Cyclical Events*: Identify the recession quarters if there are at least two consecutive standardized consumption growth drops that are <-0.9.
4. *Extreme Cyclical Events*: For an extreme event (standardized consumption growth $<=-2$), if its *immediate adjacent* quarters before and/or after exhibit negative standardized growths, then the extreme event and its adjacent quarter(s) are considered as recession quarters. Given the immediate adjacent quarter after (before) the extreme event, if the next (previous) quarters with consecutive standardized consumption growth <-0.9, they are also considered recession quarters. The purpose is to capture extreme events that typically also have a buildup before and a persistent effect after.

5. *Trough Points but with Positive Growths*: Given the recessions identified in Steps 3 and 4, if there is a recession period lasting for at least three quarters and the following quarter has a positive growth rate, then this quarter is also considered a recession period. The reason is that positive growth rates could be easily obtained given the low denominator from the previous period. This step avoids abrupt regime switches which are unrealistic.

To increase the plausibility of this algorithm, I apply it to the actual consumption growth data from January 1959 to June 2014. This algorithm is able to identify seven out of the eight NBER recessions; regressing the consumption-based recession indicator on the NBER recession indicator yields a coefficient of 0.9038 (SE=0.0507), which is statistically close to 1; without Step 5, the regression coefficient is 0.8795 (SE=0.0558). The plot below compares the consumption-based recessions (dashed; 1=recession, 0=non-recession) and the NBER recession indicator (solid; -1=recession, 0=non-recession).

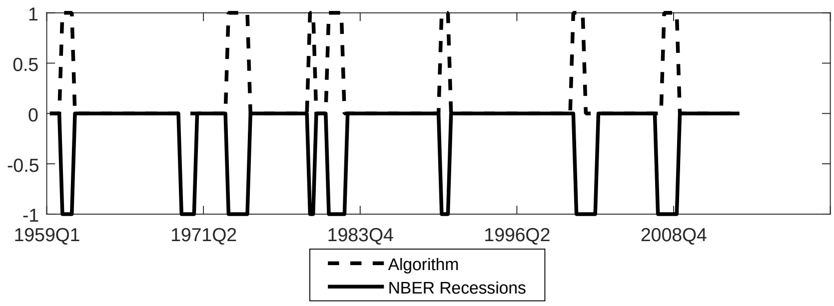


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Table 1: The conditional covariance between market returns and consumption growth.

This table presents the parameter estimates of the dynamic dependence model of market returns and consumption growth in Panel A and how the conditional second moments vary with the business cycle in Panel B. Δc_{t+1} denotes the log consumption growth, and r^m_{t+1} denotes the log market return. In Panel A, the conditional variance models are described in Section 2.1.1 (see the model selection results in Table IA1 of the Internet Appendix); the conditional correlation model is described in Section 2.1.2. In Panel B, $\sigma_t(x_{t+1})$ denotes the conditional volatility of variable x_{t+1}; $Corr_t(x_{t+1}, y_{t+1})$, the conditional correlation between variables x_{t+1} and y_{t+1}; $Cov_t(x_{t+1}, y_{t+1})$, the conditional covariance; $\beta_t(x_{t+1}, y_{t+1}) = \frac{Cov_t(x_{t+1},y_{t+1})}{\sigma_t^2(y_{t+1})}$, the conditional beta. In the first two columns, values under "No Recession" ("Recession") report the average conditional moment estimates during non-recession (recession) periods. Sample averages and standard errors of conditional covariance are scaled up by 10^5 for reporting purposes. The third column reports the t-statistic of the difference between recession and non-recession estimates. The fourth column provides interpretations of cyclicality based on the t-statistics. Standard errors are reported in parentheses; values in bold (italic) are statistically different from zero at the 5% (10%) significance level. N=665 months (1959/02∼2014/06).

Panel A. Parameter estimates					
1. Conditional variance of Δc_{t+1}, GED-GARCH-q_t					
$\sqrt{h}$	α	β	τ	ν	
0.00316	0.01286	**0.98558**	**1.47078**	**0.04285**	
(fix)	(0.07037)	(0.07095)	(0.34415)	(0.00993)	
2. Conditional variance of r^m_{t+1}, BEGE-GARCH-q_t					
$\overline{hp}$	σ_{hp}	α_{hp}	β_{hp}		
0.42390	**0.01948**	**0.01989**	**0.92476**		
(0.16461)	(0.00140)	(0.00496)	(0.01822)		
$\overline{hn}$	σ_{hn}	α_{hn}	β_{hn}	ν	
12.76012	**0.01767**	**0.16076**	**0.80066**	**0.17350**	
(0.43306)	(0.00245)	(0.00802)	(0.00941)	(0.02737)	
3. Conditional correlation between r^m_{t+1} and Δc_{t+1}, DCC-q_t					
$\overline{Q}_{12}$	α_{12}	β_{12}	ν		
0.15794	0.01713	**0.94632**	**-0.11526**		
(fix)	(0.01205)	(0.01897)	(0.03392)		

Panel B. Average conditional second moments and the state of the economy				
	No Recession:	Recession:	t-statistic: (R-NoR)	Cyclicality:
$\sigma_t(\Delta c_{t+1})$	**0.00304**	**0.00319**	2.18	Countercyclical
	(0.00002)	(0.00002)		
$\sigma_t(r^m_{t+1})$	**0.04062**	**0.05789**	37.94	Countercyclical
	(0.00031)	(0.00082)		
$Corr_t(r^m_{t+1}, \Delta c_{t+1})$	**0.16509**	**0.10994**	-9.66	Procyclical
	(0.00215)	(0.00247)		
$Cov_t(r^m_{t+1}, \Delta c_{t+1})(\times 10^5)$	**2.03297**	**2.02373**	-0.31	Weakly procyclical
	(0.03309)	(0.04704)		
$\beta_t(r^m_{t+1}, \Delta c_{t+1})$	**2.26771**	**2.10475**	-1.62	Weakly procyclical
	(0.03711)	(0.05589)		

Table 2: The decomposition.

This table presents the parameter estimates of the dynamic dependence model of market return components and consumption growth in Panel A and how the conditional second moments vary with the business cycle in Panel B. Δd_{t+1} denotes the log dividend growth. In Panel A, the conditional variance models are described in Section 2.1.1 (see the model selection results in Table IA2 of the Internet Appendix); the conditional correlation model is described in Section 2.1.2. Other table details can be found in Table 1.

Panel A. Parameter estimates						
1. Conditional variance of Δd_{t+1}, BEGE-hp_t-GARCH-q_t						
$\overline{hn}$	σ_{hn}	$\overline{hp}$	σ_{hp}	α_{hp}	β_{hp}	ν
4.08790	**0.00398**	**2.88375**	**0.00681**	**0.27713**	**0.68989**	**-0.20926**
(0.28486)	(0.00013)	(0.08248)	(0.00043)	(0.03826)	(0.02163)	(0.06982)
2. Conditional variance of $r^m_{t+1} - \Delta d_{t+1}$, BEGE-$hn_t$-GARCH-$q_t$						
$\overline{hp}$	σ_{hp}	$\overline{hn}$	σ_{hn}	α_{hn}	β_{hn}	ν
0.56790	**0.02503**	**8.60889**	**0.01776**	**0.10984**	**0.83773**	**0.22418**
(0.10152)	(0.00249)	(0.76218)	(0.00073)	(0.00228)	(0.00377)	(0.00947)
3. Conditional correlation between Δd_{t+1} and Δc_{t+1}, DCC-q_t						
$\overline{Q}_{12}$	α_{12}	β_{12}	ν			
0.01411	0.04295	**0.34287**	*-0.18957*			
(fix)	(0.04630)	(0.16937)	(0.10756)			
4. Conditional correlation between $r^m_{t+1} - \Delta d_{t+1}$ and Δc_{t+1}, DCC-q_t						
$\overline{Q}_{12}$	α_{12}	β_{12}	ν			
0.14887	0.01603	**0.95239**	**-0.10876**			
(fix)	(0.01066)	(0.01763)	(0.03278)			
Panel B. Average conditional second moments and the state of the economy						
		No Recession:	Recession:	t-statistic: (R-NoR)	Cyclicality:	
$\sigma_t(\Delta d_{t+1})$		**0.01182**	**0.00978**	-8.02	Procyclical	
		(0.00009)	(0.00004)			
$Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$		**0.01512**	**0.00689**	-2.29	Procyclical	
		(0.00151)	(0.00150)			
$Cov_t(\Delta d_{t+1}, \Delta c_{t+1})(\times 10^5)$		**0.05084**	**0.01405**	-2.09	Procyclical	
		(0.00627)	(0.00501)			
$\beta_t(\Delta d_{t+1}, \Delta c_{t+1})$		**0.05751**	**0.01720**	-1.87	Procyclical	
		(0.00782)	(0.00500)			
$\sigma_t(r^m_{t+1} - \Delta d_{t+1})$		**0.04125**	**0.05752**	28.51	Countercyclical	
		(0.00020)	(0.00022)			
$Corr_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$		**0.15538**	**0.10813**	-7.45	Procyclical	
		(0.00210)	(0.00224)			
$Cov_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})(\times 10^5)$		**1.97974**	**2.03712**	1.65	Countercyclical	
		(0.03301)	(0.04297)			
$\beta_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$		**2.22032**	**2.20144**	-0.09	Weakly procyclical	
		(0.03772)	(0.05550)			

Table 3: Closeness tests.

This table examines the closeness between the direct and indirect empirical estimates of the three return-consumption conditional comovement moments. In light of the identity of Eq. (1), the direct and indirect comovement estimates are expressed as follows:

⋄ Direct (Table 1)	⋄ Indirect (Table 2)
(1) Conditional Correlation:	
$Corr_t(r^m_{t+1}, \Delta c_{t+1})$	$\frac{\sigma_t(\Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(\Delta d_{t+1}, \Delta c_{t+1}) + \frac{\sigma_t(r^m_{t+1} - \Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$
(2) Conditional Covariance:	
$Cov_t(r^m_{t+1}, \Delta c_{t+1})$	$Cov_t(\Delta d_{t+1}, \Delta c_{t+1}) + Cov_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$
(3) Conditional Beta:	
$\beta_t(r^m_{t+1}, \Delta c_{t+1})$	$\beta_t(\Delta d_{t+1}, \Delta c_{t+1}) + \beta_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$

Panel A regresses the indirect comovement estimates on the direct ones, which constitutes a correlation test. The coefficient estimate, its 95% confidence interval, and the test result interpretation are reported in the three columns, respectively; under the null, the projection coefficient is statistically close to 1. Panel B tests whether sample moments are statistically close using the direct and the indirect comovement estimates; *** (**) indicates that the sample moment calculated using the direct estimates is within 1.645 (1.96) standard deviations of that calculated using the indirect estimates. In both panels, standard errors are reported in parentheses; in particular, standard errors in Panel B are bootstrapped standard errors (for 1000 times); values in bold (italic) are statistically different from zero at the 5% (10%) significance level. N=665 months (1959/02∼2014/06).

Panel A. Closeness test by projections			
	Coef.	95% Confidence Interval	Result
Conditional correlation	**0.9761**	[0.9521, 1.0001]	Fail to reject
	(0.0122)		
Conditional covariance	**0.9825**	[0.9608, 1.0041]	Fail to reject
	(0.0110)		
Conditional beta	**1.0138**	[0.9922, 1.0353]	Fail to reject
	(0.0110)		

Panel B. Closeness test by sample moments						
	Mean		Standard Deviation		Scaled Skewness	
	Indirect	Direct	Indirect	Direct	Indirect	Direct
Conditional correlation	**0.1570**	0.1565***	**0.0599**	0.0596***	0.0231	-0.1041**
	(0.0023)		(0.0015)		(0.0739)	
Conditional covariance	**2.0375**	2.0317***	**0.9147**	0.9110***	**0.3539**	0.2011**
	(0.0355)		(0.0226)		(0.0845)	
Conditional beta	**2.2692**	2.2449***	**1.0797**	1.0384***	**0.9694**	0.9147***
	(0.0417)		(0.0363)		(0.0938)	

Table 4: Seven extant consumption-based asset pricing models.

This table summarizes the abilities of seven well-cited variants of habit-formation (Habit) and long-run risk (LRR) models in the consumption-based asset pricing literature to fit the eight stylized facts established in Section 2: (1) CC1999: Campbell and Cochrane (1999, JPE); (2) BEX2009: Bekaert, Engstrom, and Xing (2009, JFE); (3) BE2017: Bekaert and Engstrom (2017, JPE); (4) BY2004: Bansal and Yaron (2004, JF); (5) BTZ2009: Bollerslev, Tauchen, and Zhou (2009, RFS); (6) BKY2012: Bansal, Kiku, and Yaron (2012, CFR); (7) SSY2015: Segal, Shaliastovich, and Yaron (2015, JFE). Column "Data" refers to the stylized facts presented in Tables 1 and 2. "Const." indicates that the moment is constant; "Counter-," countercyclical; and "Pro-," procyclical. Note that the cyclicality of a conditional moment in all these models can be analytically identified given the sign of its correlation with consumption growth within the model (i.e., the only macro variable).

	Data	(1) CC1999 Habit	(2) BEX2009 Habit	(3) BE2017 Habit	(4) BY2004 LRR	(5) BTZ2009 LRR	(6) BKY2012 LRR	(7) SSY2015 LRR
(a) $Var_t(\Delta c_{t+1})$	Counter-	Const.	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)
(b) $Var_t(\Delta d_{t+1})$	Pro-	Const.	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Const.
(c) $Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$	Pro-	0.2	Const.	Unclear	0	1	Const.	0
(d) $Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$	Pro-	Const.	Counter-	Counter-	0	Counter- (⊛)	Counter- (⊛)	0
(e) $\beta_t(\Delta d_{t+1}, \Delta c_{t+1})$	Pro-	Const.	Const.	Pro- (∗)	0	Const.	Const.	0
(f) $Var_t(r^m_{t+1} - \Delta d_{t+1})$	Counter-	Counter-	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)
(g) $Var_t(r^m_{t+1})$	Counter-	Counter-	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)
(h) $Cov_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$	Counter-	Counter-	Counter-	Counter-	0	0	0	Counter- (⊛)
(Duffee). $Cov_t\,(r^m_{t+1}, \Delta c_{t+1})$	Pro-	Counter-	Counter-	Counter-	0	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)

(∗) Procyclical when the scale parameter of bad uncertainty shock in the total consumption shock (σ_{cn}) is greater than the scale parameter of bad uncertainty in dividend (σ_{dn}) in BE2017.
(⊛) Countercyclical when the time-varying consumption volatility is modeled to be countercyclical; note that the time-varying volatility is a crucial feature of LRR models; however, the LRR models do not imply countercyclical volatility because the volatility shock and the consumption shock are assumed uncorrelated.

Table 5: The new DGP for the joint dividend-consumption dynamics.

Consumption growth and dividend growth have the following joint dynamics:

$$\begin{aligned}
\Delta c_{t+1} &= \bar{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, \\
n_{t+1} &= (1-\phi_n)\overline{n} + \phi_n n_t + \sigma_{nn} \widetilde{\omega}_{n,t+1}, \\
\Delta d_{t+1} &= \overline{d} + \phi_d \left(V_{c,t} - \overline{V}_c\right) + b_t \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, \\
b_{t+1} &= (1-\phi_b)\overline{b} + \phi_b b_t + \lambda_b \sigma_c \widetilde{\omega}_{c,t+1}, \\
V_{c,t} &= \sigma_c^2 + \sigma_n^2 n_t, \\
\overline{V}_c &= \sigma_c^2 + \sigma_n^2 \overline{n},
\end{aligned}$$

where the fundamental shock $\widetilde{\omega}_{c,t+1} \sim N(0,1)$, the event shock $\widetilde{\omega}_{n,t+1} \sim \Gamma(n_t,1) - n_t$, and the dividend-specific shock $\widetilde{\omega}_{d,t+1} \sim \Gamma(V_d,1) - V_d$. The DGP estimation adopts a two-step procedure and uses the AR(3)-centered consumption growth and the original dividend growth as Δc_{t+1} and Δd_{t+1} (see details in Appendix Appendix B). Panels A and B present the estimation results. Panel C reports the cyclicality tests; rows "$b(I_{NBER,t})$" report the regression coefficients of a state variable or an implied conditional second moment on the NBER recession indicator. Standard errors are shown in parentheses. Values in bold (italic) are statistically different from zero at the 5% (10%) significance level. N=665 months (1959/02∼2014/06).

Panel A. Estimation results, consumption				**Panel B. Estimation results, dividend**			
	Δc_{t+1}		n_{t+1}		Δd_{t+1}		b_{t+1}
$\bar{c}$	**0.0025**	$\bar{n}$	**0.3742**	$\bar{d}$	**0.0015**	$\bar{b}$	0.0944
	(0.0001)		(0.1609)		(0.0004)		(0.1612)
σ_c	**0.0029**	ϕ_n	**0.9500**	ϕ_d	**-630.8768**	ϕ_b	**0.3159**
	(0.0001)		(0.0264)		(225.7119)		(0.1561)
σ_n	**-0.0023**	σ_{nn}	**0.2772**	σ_d	**1.23E-04**	λ_b	**59.9163**
	(0.0005)		(0.1027)		(3.36E-06)		(5.4008)
				V_d	**8933.5172**		
					(488.4230)		

Panel C. Confirming the cyclicalities

	State Variables		*Fact (a)*	*Fact (b)*
	n_{t+1}	b_{t+1}	$\sigma_t(\Delta c_{t+1})$	$\sigma_t(\Delta d_{t+1})$
$b(I_{NBER,t})$	**0.5926**	**-0.1240**	**4.28E-04**	*-4.35E-06*
	(0.0354)	(0.0190)	(2.68E-05)	(2.59E-06)
	Fact (c)		*Fact (d)* ($\times 10^5$)	*Fact (e)*
	$Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$		$Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$	$\beta_t(\Delta d_{t+1}, \Delta c_{t+1})$
$b(I_{NBER,t})$	**-0.0282**		**-0.1033**	**-0.1041**
	(0.0042)		(0.0159)	(0.0155)

Table 6: Properties of the filtered DGP shocks.

This table presents statistical properties of the three filtered DGP shocks. Panel A reports the correlation between the monthly and quarterly shocks and business cycle indicators. Monthly shocks are obtained from the estimation, quarterly shocks are obtained using the sum of monthly shocks within the quarter, and business cycle indicators include the NBER recession indicator and the detrended $\widehat{cay}$ variable from Lettau and Ludvigson (2001). Panel B presents the summary statistics of the three filtered monthly shocks. Bootstrapped standard errors are reported in parentheses in both panels. Values in bold (italic) are statistically different from zero at the 5% (10%) significance level. N=665 months (1959/02∼2014/06).

Panel A. Correlation w/ the business cycle				**Panel B. Summary statistics, monthly**			
	$\widetilde{\omega}_c$	$\widetilde{\omega}_n$	$\widetilde{\omega}_d$		$\widetilde{\omega}_c$	$\widetilde{\omega}_n$	$\widetilde{\omega}_d$
NBER, monthly	**-0.18**	**0.13**	**-0.11**	Mean	1.71E-03	2.42E-03	1.66E-04
	(0.04)	(0.04)	(0.04)		(0.04)	(0.02)	(3.62)
				Standard Deviation	**0.97**	**0.44**	**94.47**
	$\widetilde{\omega}_c^Q$	$\widetilde{\omega}_n^Q$	$\widetilde{\omega}_d^Q$		(0.03)	(0.06)	(3.87)
NBER, quarterly	**-0.27**	**0.25**	**-0.23**	Scaled Skewness	0.18	**5.44**	0.19
	(0.07)	(0.07)	(0.07)		(0.13)	(0.57)	(0.25)
$\widehat{cay}$, quarterly	**-0.22**	0.06	0.01	Excess Kurtosis	0.45	**40.52**	**2.70**
	(0.06)	(0.06)	(0.07)		(0.50)	(8.38)	(0.56)

Table 7: Non-DGP model parameter choices ($*$=annualized).

This table presents the non-DGP parameter choices and the derived parameter values. The AR(1) coefficient of s_t, ϕ_s, is obtained from the AR(1) coefficient of the monthly log valuation ratio; rf_{CC} is the constant benchmark risk free rate and is chosen to match the average real 90-day Treasury bill rate, which is proxied by changes in the log nominal 90-day Treasury index (source: CRSP) minus inflation rate (source: FRED) continuously compounded; β is the time discount parameter derived from the rf_{CC} equation. Monthly data covers the period 1959/01∼2014/06.

		Notation	Value
1.	Non-DGP parameters:		
	Curvature parameter	γ	2
$*$	s_t persistence	ϕ_s	0.9236
$*$	Risk free rate (%)	rf_{CC}	1.4854
2.	Derived parameters:		
$*$	Discount rate	β	0.9694
	Steady-state surplus consumption ratio, M(1)	$\bar{S}$	0.0559
	Maximum log surplus consumption ratio, M(1)	s_{max}	-2.3863

Table 8: Theoretical models: cyclical moments.

This table evaluates the abilities of three overlaying theoretical models to fit Facts (a)–(h) and the Duffee Puzzle. These empirical facts are established in Section 2. **Empirical Moments:** Column "Data" presents three empirical benchmarks of each fact: (1) unconditional moments using data during non-recessions ($I_{NBER} = 0$) and (2) recessions ($I_{NBER} = 1$), and (3) a regression coefficient of the DGP-implied conditional moments on the NBER recession indicator. Bootstrapped and OLS standard errors are shown in parentheses under Column "SE." The significance of testing whether the recession/non-recession sample moments is indicated next to the non-recession moment; the significance of the regression coefficient of the conditional moments is also shown. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. **Model moments:** The counterparts using the simulated datasets of the three theoretical models are shown under Columns "M(1)", "M(2)" and "M(3)". The models are solved numerically using the "series method" introduced in Wachter (2005), and are simulated for 100,000 months; see details on calibration in Section 4.3.1. All model-implied moments in this paper are calculated using the second half of a simulated dataset, i.e., 50,001-100,000. The algorithm for identifying recession periods is described in Appendix Appendix G. **Symbols:** σ (σ_t), volatility (conditional volatility); Cov (Cov_t), covariance (conditional covariance); $Corr$ ($Corr_t$), correlation (conditional correlation); β (β_t), beta (conditional beta). **Performance:** Bold (italic) values indicate that the simulation moment point estimates are within a 95% (99%) confidence interval of the empirical moments; "-" indicates that the model-implied conditional moment is constant and insensitive to a recession indicator, e.g., $Cov_t(\Delta d_{t+1}, \Delta c_{t+1}) = \bar{b}\sigma_c^2$ in M(2).

		Data	SE	M(1) Adapted Campbell & Cochrane, 1999	M(2) Adapted Bekaert & Engstrom, 2017	M(3) This Paper
	s as State Variable	-	-	Yes	Yes	Yes
	n as State Variable	-	-	No	Yes	Yes
	b as State Variable	-	-	No	No	Yes
(a)	$\sigma(\Delta c)$ ($I_{rece.} = 0$)	0.0031*	(0.0001)	**0.0032**	**0.0032**	**0.0032**
	$\sigma(\Delta c)$ ($I_{rece.} = 1$)	0.0036	(0.0002)	0.0032	**0.0035**	**0.0035**
	$\sigma_t(\Delta c_{t+1}) \sim I_{rece.,t}$	4.28E-04***	(2.68E-05)	-	2.47E-04	2.47E-04
(b)	$\sigma(\Delta d)$ ($I_{rece.} = 0$)	0.0118***	(0.0005)	**0.01158**	**0.01173**	**0.01174**
	$\sigma(\Delta d)$ ($I_{rece.} = 1$)	0.0092	(0.0008)	0.01177	0.01214	0.01218
	$\sigma_t(\Delta d_{t+1}) \sim I_{rece.,t}$	-4.35E-06*	(2.59E-06)	-	-	**-4.00E-06**
(c)	$Corr(\Delta d, \Delta c)$ ($I_{rece.} = 0$)	0.0303	(0.0388)	**0.0223**	**0.0208**	**0.0218**
	$Corr(\Delta d, \Delta c)$ ($I_{rece.} = 1$)	-0.0013	(0.0388)	**0.0223**	**0.0203**	**0.0054**
	$Corr_t(\Delta d_{t+1}, \Delta c_{t+1}) \sim I_{rece.,t}$	-0.0282***	(0.0042)	-	-0.0012	**-0.0348**
(d)	$Cov(\Delta d, \Delta c)$ ($I_{rece.} = 0$) ($\times 10^5$)	0.1110	(0.1424)	**0.0818**	**0.0770**	**0.0809**
	$Cov(\Delta d, \Delta c)$ ($I_{rece.} = 1$) ($\times 10^5$)	-0.0044	(0.1286)	**0.0843**	**0.0868**	**0.0232**
	$Cov_t(\Delta d_{t+1}, \Delta c_{t+1}) \sim I_{rece.,t}$ ($\times 10^5$)	-0.1033***	(0.0159)	-	-	**-0.1242**
(e)	$\beta(\Delta d, \Delta c)$ ($I_{rece.} = 0$)	0.1155	(0.1482)	**0.0817**	**0.0770**	**0.0810**
	$\beta(\Delta d, \Delta c)$ ($I_{rece.} = 1$)	-0.0034	(0.1003)	**0.0815**	**0.0700**	**0.0187**
	$\beta_t(\Delta d_{t+1}, \Delta c_{t+1}) \sim I_{rece.,t}$	-0.1041***	(0.0155)	-	-0.0075	**-0.1319**
(f)	$\sigma(r^m - \Delta d)$ ($I_{rece.} = 0$)	0.0413***	(0.0020)	0.0146	**0.0423**	**0.0419**
	$\sigma(r^m - \Delta d)$ ($I_{rece.} = 1$)	0.0665	(0.0066)	0.0138	**0.0537**	*0.0509*
(g)	$\sigma(r^m)$ ($I_{rece.} = 0$)	0.0400***	(0.0020)	0.0188	**0.0423**	**0.0420**
	$\sigma(r^m)$ ($I_{rece.} = 1$)	0.0652	(0.0061)	0.0183	**0.0539**	0.0485
(h)	$Cov(r^m - \Delta d, \Delta c)$ ($I_{rece.} = 0$) ($\times 10^5$)	1.7436*	(0.4925)	4.0956	3.1658	*2.9983*
	$Cov(r^m - \Delta d, \Delta c)$ ($I_{rece.} = 1$) ($\times 10^5$)	3.3938	(0.9146)	**4.1321**	**3.9563**	**3.8239**
(Duffee)	$Cov(r^m, \Delta c)$ ($I_{rece.} = 0$) ($\times 10^5$)	1.8546	(0.4767)	4.1775	3.2428	*3.0792*
	$Cov(r^m, \Delta c)$ ($I_{rece.} = 1$) ($\times 10^5$)	3.3894	(0.8966)	**4.2165**	**4.0431**	**3.8470**

Table 9: Theoretical models: unconditional moments.

This table presents 17 unconditional moments from empirical and simulated datasets. Details on data, models, and simulations are described in Table 8. Bootstrapped standard errors are shown in parentheses. Bold (italic) values indicate that the simulation moment point estimates are within a 95% (99%) confidence interval of the empirical moments.

	Data	SE	M(1) Adapted Campbell & Cochrane, 1999	M(2) Adapted Bekaert & Engstrom, 2017	M(3) This Paper
s as State Variable	-	-	Yes	Yes	Yes
n as State Variable	-	-	No	Yes	Yes
b as State Variable	-	-	No	No	Yes
$E(\Delta c)$	0.0025	(0.0001)	**0.0025**	**0.0025**	**0.0025**
$\sigma(\Delta c)$	0.0032	(0.0001)	**0.0032**	**0.0032**	**0.0032**
$Skew(\Delta c)$	-0.1292	(0.1419)	**-0.2658**	**-0.2707**	**-0.2707**
$xKurt(\Delta c)$	0.7779	(0.3553)	**0.5342**	**0.8354**	**0.8354**
Heteroskedastic Δc Innovations	Yes		No	**Yes**	**Yes**
$E(\Delta d)$	0.0015	(0.0005)	**0.0015**	**0.0015**	**0.0015**
$\sigma(\Delta d)$	0.0116	(0.0005)	**0.0116**	**0.0117**	**0.0118**
$Skew(\Delta d)$	0.2268	(0.2478)	**0.0285**	**0.0117**	**0.0122**
$xKurt(\Delta d)$	2.7560	(0.5656)	-0.0152	-0.0052	-0.0060
Heteroskedastic Δd Innovations	Yes		No	No	**Yes**
$Corr(\Delta d_{t+1}, \Delta c_{t+1})$	0.0569	(0.0343)	**0.0224**	**0.0225**	**0.0225**
$Cov(\Delta d_{t+1}, \Delta c_{t+1})(\times 10^5)$	0.2140	(0.1341)	**0.0831**	**0.0847**	**0.0849**
$\beta(\Delta d_{t+1}, \Delta c_{t+1})$	0.2052	(0.1278)	**0.0812**	**0.0824**	**0.0825**
$\sigma(r^m - \Delta d)$	0.0458	(0.0019)	0.0147	**0.0428**	**0.0421**
$\sigma(r^m)$	0.0448	(0.0019)	0.0189	**0.0427**	**0.0422**
$Cov(r^m - \Delta d, \Delta c)$ $(\times 10^5)$	2.2682	(0.5574)	4.1826	**3.2568**	**3.0889**
$Cov(r^m, \Delta c)$ $(\times 10^5)$	2.4822	(0.5624)	4.2657	**3.3416**	**3.1738**

Table 10: Theoretical models: conventional asset price statistics ($*$=annualized).

This table presents ten unconditional moments of asset prices from actual and simulated datasets. Bold (italic) values indicate that the simulation moment point estimates are within a 95% (99%) confidence interval of the empirical moments.

		Data	SE	M(1) Adapted Campbell & Cochrane, 1999	M(2) Adapted Bekaert & Engstrom, 2017	M(3) This Paper
	s as State Variable	-	-	Yes	Yes	Yes
	n as State Variable	-	-	No	Yes	Yes
	b as State Variable	-	-	No	No	Yes
$*$	$E(r^m - rf), \%$	4.7964	(2.0829)	**3.8751**	**6.2414**	**5.6524**
$*$	$\sigma(r^m - rf), \%$	15.4516	(0.6197)	6.4629	**14.9684**	**14.7234**
	$\exp[E(pd)]$	35.992	(0.5461)	25.8418	17.2668	17.5131
	$\sigma(pd)$	0.3847	(0.0895)	0.1090	**0.2429**	**0.2594**
$*$	$ac(pd)$	0.9236	(0.0557)	**0.9063**	**0.8751**	**0.8757**
	Sharpe Ratio	0.3276	(0.1501)	**0.5992**	**0.4236**	**0.3888**
	Scaled Skewness	-0.7932	(0.2592)	0.1515	-0.1816	-0.1453
	Excess Kurtosis	2.6386	(1.2713)	**0.3318**	**0.5579**	**0.4870**
$*$	$E(rf), \%$	1.4854	(0.1525)	*1.1159*	**1.3608**	**1.3608**
$*$	$\sigma(rf), \%$	0.9895	(0.0428)	0.0348	0.0450	0.0450

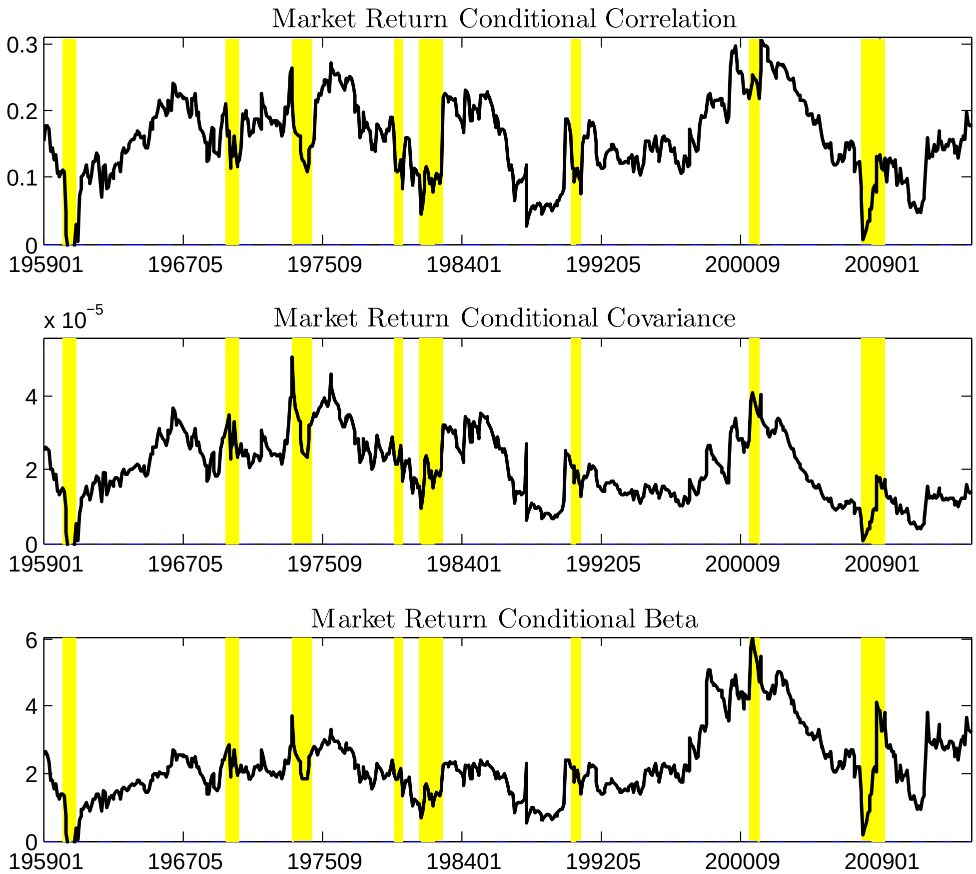

Figure 1: Empirical model: the conditional comovement between market returns and consumption growth.

The figure depicts the dynamics of the estimated return-consumption conditional correlation $Corr_t(r^m_{t+1}, \Delta c_{t+1})$ (top), conditional covariance $Cov_t(r^m_{t+1}, \Delta c_{t+1})$ (middle), and conditional beta $\beta_t(r^m_{t+1}, \Delta c_{t+1})$ (bottom); estimation details are reported in Table 1. In the second and third plots, the spike around 1987 corresponds to the stock market Black Monday in October 1987, which caused extreme price movements. The shaded regions are the NBER recession months from the NBER website.

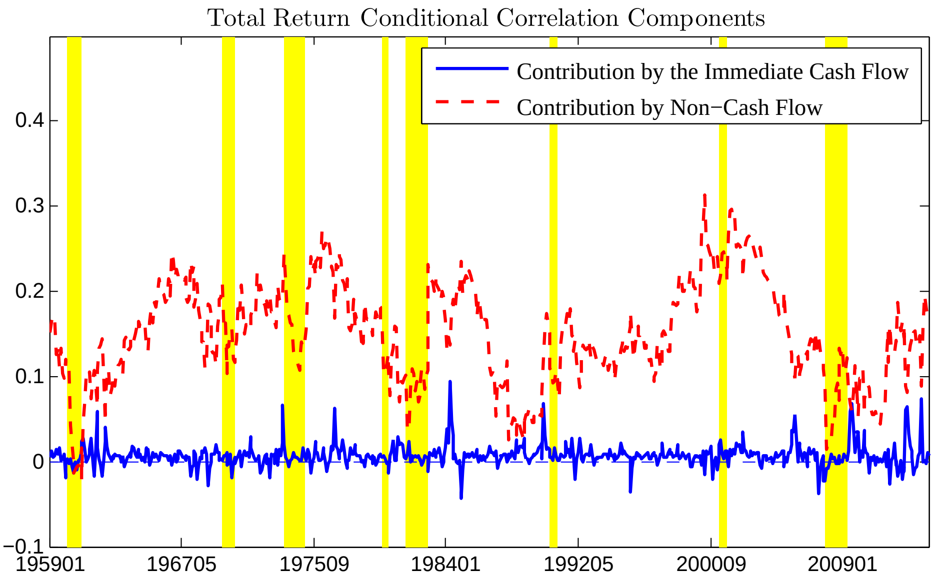

Figure 2: Empirical model: the decomposition In the correlation space.

In light of the following identity,

$$Corr_t(r^m_{t+1}, \Delta c_{t+1}) = \frac{\sigma_t(\Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(\Delta d_{t+1}, \Delta c_{t+1}) + \frac{\sigma_t(r^m_{t+1} - \Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1}),$$

this figure shows the dynamics of the immediate cash flow component (in solid blue) of the total return-consumption conditional correlation, $\frac{\sigma_t(\Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$, and the remaining non-cash flow component (in dashed red), $Corr_t(r^m_{t+1}, \Delta c_{t+1}) - \frac{\sigma_t(\Delta d_{t+1})}{\sigma_t(r^m_{t+1})} Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$. The total market return correlation estimates are depicted in Fig. 1. The non-cash flow component depicted in this plot uses the difference between the return conditional correlation and its immediate cash flow component; Figure IA2 of the Internet Appendix compares this indirect non-cash flow measure with a direct measure, and they are correlated at 0.99. This figure and Section 2.3.3 discuss the decomposition in the correlation space for interpretation and illustration purposes. Figure IA3 of the Internet Appendix presents this decomposition in the covariance space. The shaded regions are the NBER recession months from the NBER website.

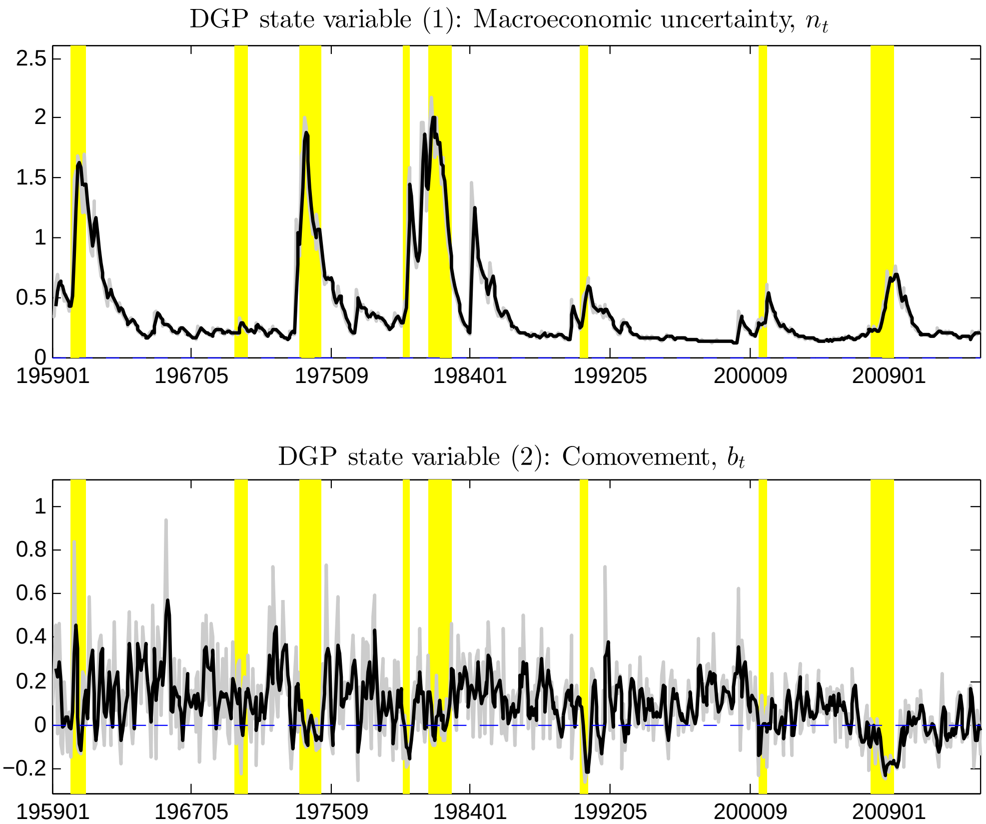

Figure 3: DGP: state variables.

In both plots, gray lines depict the monthly estimates of the latent state variables, and the overlaying black lines depict their three-month moving averages. The countercyclical macroeconomic uncertainty state variable n_t (top) is estimated from a filtration-based maximum likelihood estimation methodology developed by Bates (2006), and the procyclical dividend-consumption comovement state variable b_t (bottom) is estimated using MLE; detailed estimation procedure is provided in Appendix Appendix B; detailed estimation results are shown in Table 5. The monthly n_t (b_t) estimates exhibit a significant correlation of 0.545 (-0.245) with the NBER recession indicator. The augmented Dickey-Fuller test statistic of the monthly n_t (b_t) estimates rejects the unit root null hypothesis with a test statistic of -4.298 (-18.775). The shaded regions are the NBER recession months from the NBER website.

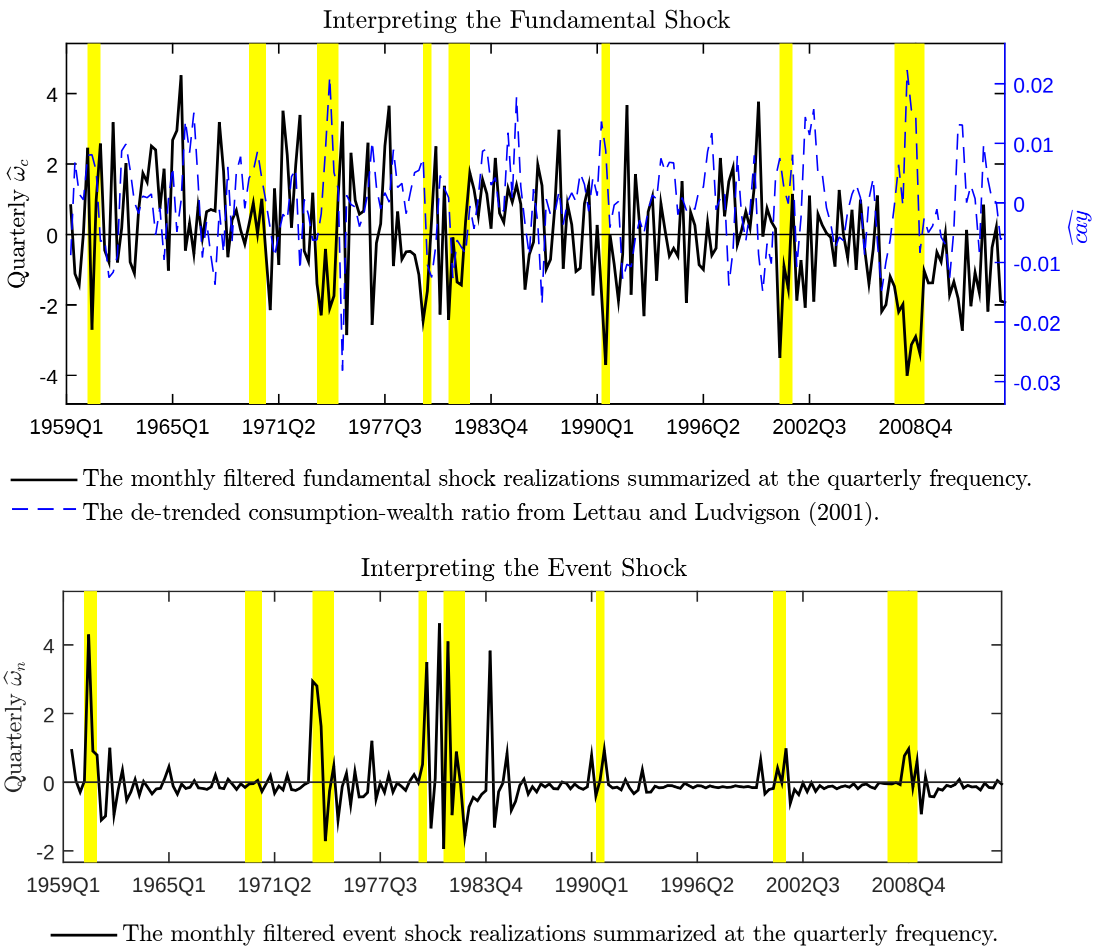

Figure 4: DGP: economic interpretations of consumption shocks.

This figure provides graphical evidence for potential economic interpretations of the filtered fundamental and event consumption shocks. The top plot provides a quarter-to-quarter comparison between the fundamental shock (in solid black; left axis) and the de-trended consumption-wealth ratio (in dashed blue; right axis) from Lettau and Ludvigson (2001), or $\widehat{cay}$; their correlation is significant and negative (-0.22). The bottom plot depicts the quarterly event shock realizations; its correlation with the NBER recession indicator is significant and positive (0.25). The shaded regions are the NBER recession quarters from the NBER website.