NOT FOR PUBLICATION

Internet Appendix for "Procyclicality of the comovement between dividend growth and consumption growth"

Nancy R. Xu*

April 2, 2020

Table IA1 presents the model selection results of the conditional variance models for consumption growth and market returns

Table IA2 presents the model selection results of the conditional variance models for return components (using the same model class as market returns)

Table IA3 presents how the estimated return-consumption conditional comovements vary with business cycle instruments: (1) the NBER recession indicator, as used in my paper, and (2) *cay* from Lettau and Ludvigson (2001), as used in Duffee (2005)

Table IA4 presents the unconditional correlations using raw data, residuals, and standardized residuals

Figure IA1 depicts the dynamics of the conditional volatilities of consumption growth, market returns, dividend growth, and the difference between market returns and dividend growth from the empirical model

Figure IA2 depicts the dynamics of the valuation part of the return-consumption conditional correlation using the direct and indirect estimates

Figure IA3 depicts the decomposition in the covariance space

*Boston College, Carroll School of Management, 140 Commonwealth Avenue, Chestnut Hill, MA, 02467, USA. Please feel free to email me if you have any questions: nancy.xu@bc.edu

Table IA1: Conditional Variance Models for Consumption Growth and Market Returns.

This table presents the model selection results of the conditional variance models for consumption growth and market returns. Denote $\widetilde{\epsilon}_{t+1}$ as the residual and h_t as its conditional variance. **"Unconditional-q_t":** The first model assumes that the residuals follow a conditional Gaussian distribution, $\widetilde{\epsilon}_{t+1} \sim N(0, h_t)$, and the conditional variance h_t is a two-state process,

$$h_t = \overline{h}\,(1 + q_t)\,, \qquad \text{(eq: IA1)}$$

$$q_t = \nu SNBER_t, \qquad \text{(eq: IA2)}$$

where $\overline{h}$ denotes the pre-determined unconditional variance; q_t is a cyclical variable and is modeled as a multiple of the standardized NBER recession indicator; ν is an unknown parameter. **"GARCH-q_t", "GED-GARCH-q_t":** The second and third conditional variance models introduce this cyclical component into a generalized autoregressive conditional heteroskedastic process as its long-run mean "$\overline{h}\,(1 + q_t)$":

$$h_t = \overline{h}\,(1 + q_t) + \alpha\left[\widetilde{\epsilon}_t^2 - \overline{h}\,(1 + q_{t-1})\right] + \beta\left[h_{t-1} - \overline{h}\,(1 + q_{t-1})\right], \qquad \text{(eq: IA3)}$$

where $\alpha + \beta < 1$, $\alpha > 0$, $\beta > 0$; q_t is modeled similarly. The second model assumes a conditional Gaussian distribution, while the third model assumes a symmetric leptokurtic conditional generalized error distribution (GED) with an unknown shape parameter τ. **"BEGE-GARCH-q_t":** The fourth model is in the same class of the "Bad Environment-Good Environment" (BEGE) model by Bekaert, Engstrom, and Ermolov (2015). The residuals follow a composite distribution of two centered gamma shocks that independently and separately govern the left- and right-tail behaviors. The composite residual is assumed as $\widetilde{\epsilon}_{t+1} = \sigma_{hp}\widetilde{\omega}_{hp,t+1} - \sigma_{hn}\widetilde{\omega}_{hn,t+1}$, where $\widetilde{\omega}_{hp,t+1} \sim \widetilde{\Gamma}(hp_t, 1)$, $\widetilde{\omega}_{hn,t+1} \sim \widetilde{\Gamma}(hn_t, 1)$, shape parameters $hp_t > 0$ and $hn_t > 0$ for all t, and scale parameters $\sigma_{hp} > 0$ and $\sigma_{hn} > 0$. The left- and right-tail shape parameters, hn_t and hp_t, are modeled as follows:

$$hn_t = \overline{hn}\,(1 + q_t) + \alpha_{hn}\left[\frac{\widetilde{\epsilon}_t^2}{2\sigma_{hn}^2} - \overline{hn}\,(1 + q_{t-1})\right] + \beta_{hn}\left[hn_{t-1} - \overline{hn}\,(1 + q_{t-1})\right], \qquad \text{(eq: IA4)}$$

$$hp_t = \overline{hp}\,(1 + q_t) + \alpha_{hp}\left[\frac{\widetilde{\epsilon}_t^2}{2\sigma_{hp}^2} - \overline{hp}\,(1 + q_{t-1})\right] + \beta_{hp}\left[hp_{t-1} - \overline{hp}\,(1 + q_{t-1})\right], \qquad \text{(eq: IA5)}$$

where $\overline{hn}$ and $\overline{hp}$ are now unknown parameters because they depend on the value of scale parameters σ_{hn} and σ_{hp}, respectively; $\alpha_{hn} + \beta_{hn} < 1$, $\alpha_{hp} + \beta_{hp} < 1$, $\alpha_{hn} > 0$, $\alpha_{hp} > 0$, $\beta_{hn} > 0$, $\beta_{hp} > 0$; q_t is modeled similarly. Given the distributional assumption, the total conditional variance is $h_t = \sigma_{hp}^2 hp_t + \sigma_{hn}^2 hn_t$. In all four models above, a positive (negative) ν estimate indicates that there is a countercyclical (procyclical) slow-moving component in the conditional variance. **"Unconditional", "GARCH", "GED-GARCH", "BEGE-GARCH":** The next four models assuming $\nu = 0$ are also estimated. The estimations for "GARCH", "GARCH-q_t", "GED-GARCH" and "GED-GARCH-q_t" use variance targeting. The chosen models among the eight models are indicated with "Use", according to the Bayesian information criteria (BIC). The estimates and robust standard errors of the cyclicality parameter ν are also reported. Values in bold are statistically different from zero at the 5% significance level. The full parameter estimates of the chosen model are presented in Table 1 of the paper. N=665 months (1959/02∼2014/06).

	Panel A. Consumption Growth						Choice
	Loglikelihood	Nparams	AIC	BIC	ν	SE(ν)	
Unconditional	2884.42	1	-5766.84	-5762.34			
GARCH	2911.44	2	-5816.87	-5803.37			
GED-GARCH	2913.30	3	-5820.59	-5807.10			
BEGE-GARCH	2919.99	8	-5823.99	-5787.99			
Unconditional-q_t	2885.92	1	-5767.84	-5758.84	**0.1014**	(0.0066)	
GARCH-q_t	2913.14	3	-5820.28	-5806.78	**0.0270**	(0.0060)	
GED-GARCH-q_t	2917.30	4	-5826.59	-5808.60	**0.0428**	(0.0099)	Use
BEGE-GARCH-q_t	2925.01	9	-5832.03	-5791.53	**0.1462**	(0.0243)	
	Panel B. Market Returns						
	Loglikelihood	Nparams	AIC	BIC	ν	SE(ν)	
Unconditional	1124.50	1	-2247.01	-2242.51			
GARCH	1145.39	2	-2286.77	-2277.77			
GED-GARCH	1157.51	3	-2309.03	-2295.53			
BEGE-GARCH	1174.76	8	-2333.53	-2297.53			
Unconditional-q_t	1148.47	1	-2294.95	-2290.45	**0.4691**	(0.0873)	
GARCH-q_t	1157.99	3	-2309.98	-2296.48	**0.4445**	(0.0850)	
GED-GARCH-q_t	1166.84	4	-2325.69	-2307.69	**0.4391**	(0.0943)	
BEGE-GARCH-q_t	1185.44	9	-2352.88	-2312.38	**0.1735**	(0.0274)	Use

Table IA2: Conditional Variance Models for Return Components.

This table presents the model selection results of the conditional variance models for return components chosen from the same conditional variance model class of market returns (the BEGE-GARCH framework) to ensure economic consistency. Three models in the BEGE class are considered. **"BEGE-GARCH-q_t"** is as introduced in Table IA1 which allows time-varying shape parameters of both left- and right-tail gamma shocks. **"BEGE-hn_t-GARCH-q_t"** and **"BEGE-hp_t-GARCH-q_t"** are two "half" BEGE-GARCH-q_t models with time-varying left-tail and right-tail shape parameters hn_t and hp_t, respectively; for instance, with a constant right-tail shape parameter at $\overline{hp}$, the conditional variance in BEGE-hn_t-GARCH-q_t is $h_t = \sigma_{hp}^2 \overline{hp} + \sigma_{hn}^2 hn_t$; similar logic applies to BEGE-hp_t-GARCH-q_t. The chosen models are indicated with "Use", according to the BIC. The estimates and robust standard errors of the cyclicality parameter ν are also reported. Values in bold are statistically different from zero at the 5% significance level. The full parameter estimates of the chosen model are presented in Table 2 of the paper. N=665 months (1959/02∼2014/06).

	Panel A. Dividend Growth						Choice
	Loglikelihood	Nparams	AIC	BIC	ν	SE(ν)	
BEGE-GARCH-q_t	2083.01	9	-4148.02	-4107.52	**-0.1987**	(0.0788)	
BEGE-hn_t-GARCH-q_t	2077.23	7	-4140.45	-4108.95	**-0.2059**	(0.0545)	
BEGE-hp_t-GARCH-q_t	2081.13	7	-4148.26	-4116.76	**-0.2093**	(0.0698)	Use
	Panel B. Market Return–Dividend Growth						
	Loglikelihood	Nparams	AIC	BIC	NBER coefficient (SE)		
BEGE-GARCH-q_t	1165.01	9	-2312.01	-2271.51	**0.1977**	(0.0302)	
BEGE-hn_t-GARCH-q_t	1160.20	7	-2306.40	-2274.90	**0.2242**	(0.0095)	Use
BEGE-hp_t-GARCH-q_t	1152.10	7	-2290.20	-2258.70	**0.1926**	(0.0294)	

Table IA3: Cyclicality using Various Business Cycle Instruments.

This table presents the correlations between return-consumption conditional comovement estimates and two business cycle indicators: (1) the NBER recession indicator (1=recession; 0=non-recession), and (2) the de-trended $\widehat{cay}$ from Lettau and Ludvigson (2001). Given that $\widehat{cay}$ is available at the quarterly frequency, monthly $\widehat{cay}$ values are obtained using the last quarterly value. Values in bold are statistically different from zero at the 5% significance level; p-values are also reported with square brackets.

	$Corr_t(r^m_{t+1}, \Delta c_{t+1})$	$Cov_t(r^m_{t+1}, \Delta c_{t+1})$	$\beta_t(r^m_{t+1}, \Delta c_{t+1})$
Panel A. Sample: 1959/02-2014/06			
(1) NBER	**-0.32**	-0.04	-0.05
p-value:	[0.00]	[0.43]	[0.16]
(2) $\widehat{cay}$	**-0.13**	**-0.13**	**-0.08**
	[0.00]	[0.00]	[0.03]
Panel B. Sample: 1959/02-2001/12 (Duffee, 2005)			
(1) NBER	**-0.54**	**-0.32**	**-0.31**
	[0.00]	[0.00]	[0.00]
(2) $\widehat{cay}$	**-0.50**	**-0.48**	**-0.54**
	[0.00]	[0.00]	[0.00]

Table IA4: Unconditional Correlations between Consumption Growth and Market Return Components.

This table presents the unconditional correlations between consumption growth and other variables-of-interest. "Raw data" is described in Section 2.2 of the paper; "Residuals" are obtained by regressing the raw data on a business cycle indicator (NBER recession dummy); "Standardized residuals" are obtained by dividing the residuals with the corresponding conditional volatility estimates. Values in the last row are also used as pre-determined $\overline{Q}_{12}$s in Tables 1 and 2 of the paper.

	r^m_{t+1}	Δd_{t+1}	$r^m_{t+1} - \Delta d_{t+1}$
Raw data	0.1715	0.0569	0.1533
Residuals	0.1576	0.0242	0.1476
Standardized residuals	0.1579	0.0141	0.1489

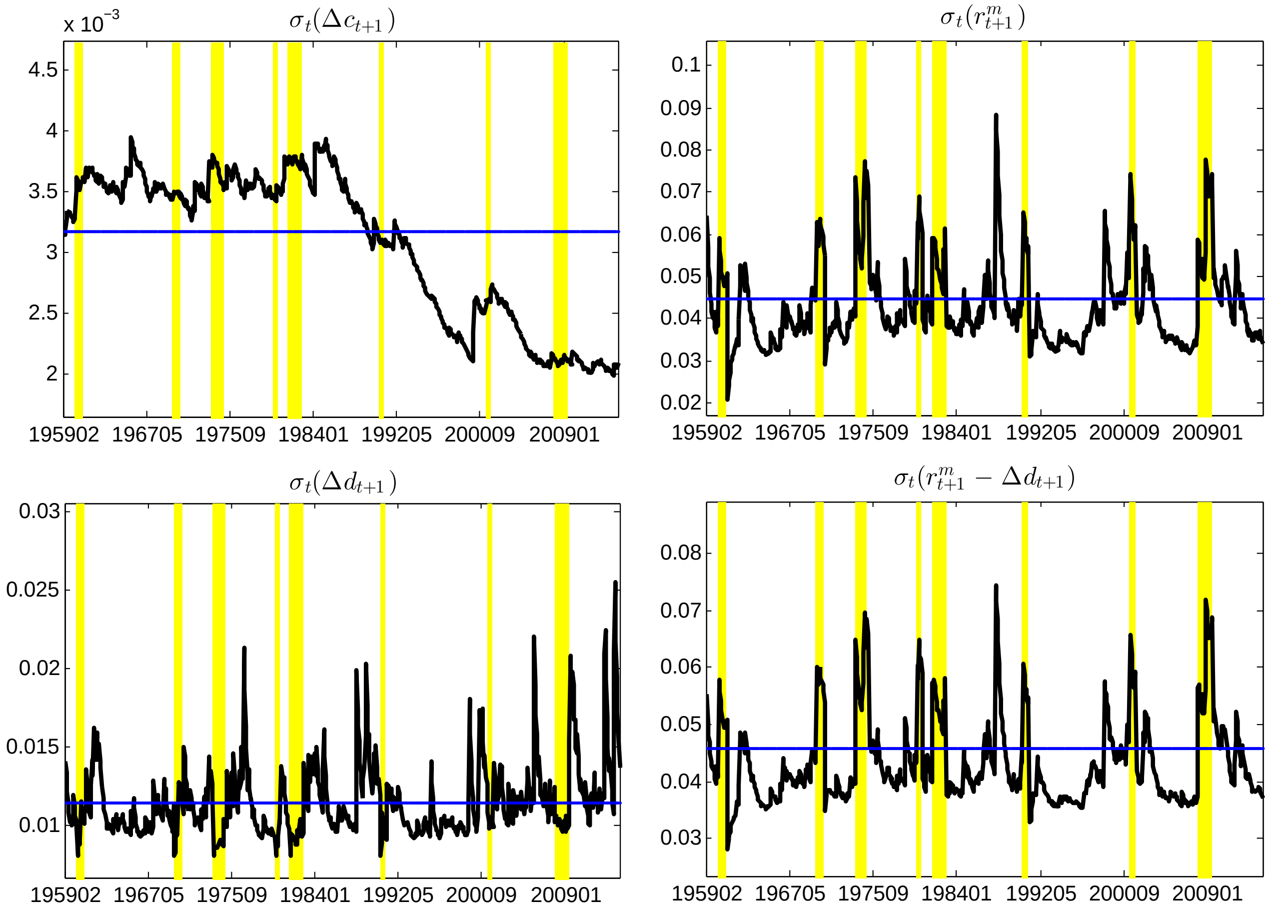

Figure IA1: Empirical Model: Conditional Volatility Estimates.

From left to right, top to bottom: conditional volatility estimates of consumption growth $\sigma_t(\Delta c_{t+1})$, market returns $\sigma_t(r^m_{t+1})$, dividend growth $\sigma_t(\Delta d_{t+1})$, and the difference between market returns and dividend growth $\sigma_t(r^m_{t+1} - \Delta d_{t+1})$. The chosen conditional variance models are shown in Tables IA1 and IA2 of this internet appendix. Horizontal lines indicate the unconditional volatilities calculated using residuals. The shaded regions are the NBER recession months from the NBER website.

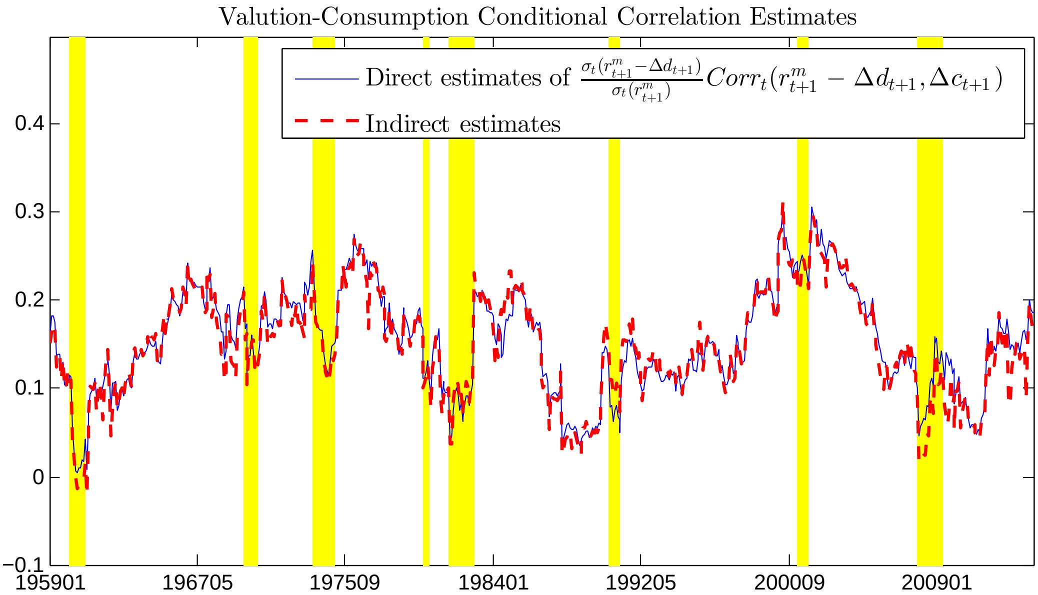

Figure IA2: Empirical Model: Direct and Indirect Estimates of the Valuation Part of the Return-consumption Conditional Correlation.

Given the identity of Equation (8) of the paper, the direct estimates of the valuation part of the return-consumption conditional correlation are obtained as $\frac{\sigma_t(r^m_{t+1}-\Delta d_{t+1})}{\sigma_t(\Delta c_{t+1})} Corr_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$, while the indirect estimates as obtained as $Corr_t(r^m_{t+1}, \Delta c_{t+1}) - \frac{\sigma_t(\Delta d_{t+1})}{\sigma_t(\Delta c_{t+1})} Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$. The indirect estimates are also plotted as the dashed red line in Figure 2 of the paper. The two series are correlated at 0.99. The shaded regions are the NBER recession months from the NBER website.

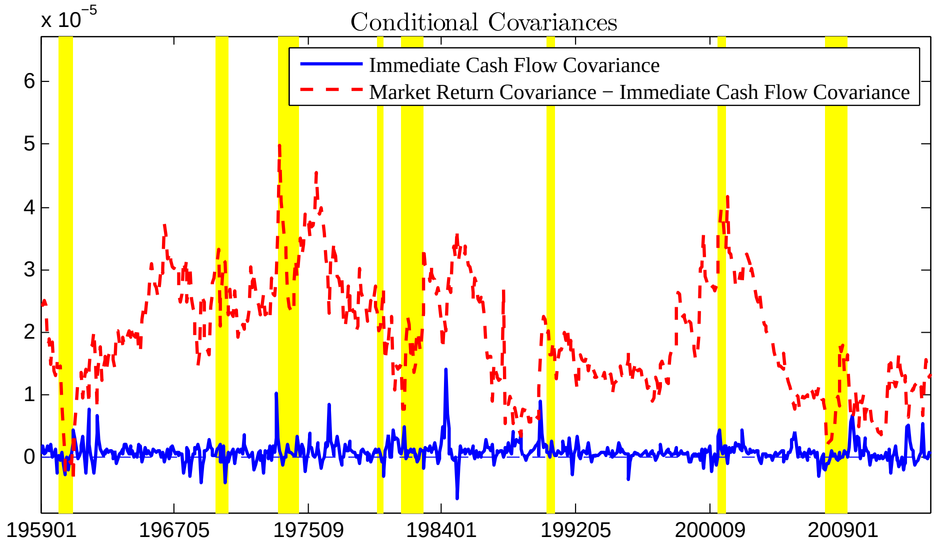

Figure IA3: Empirical Model: The Decomposition in the Covariance Space.

This figure shows the dynamics of the immediate cash flow covariance (depicted with a solid blue line) and the valuation covariance (depicted with a dashed red line). Both the return and the immediate cash flow conditional covariances are separately estimated from a cyclic DCC model; the valuation covariance is proxied as the difference between the estimates of the total return conditional covariance and the immediate cash flow covariance. The shaded regions are the NBER recession months from the NBER website. Note: Figure 2 of the paper depicts the decomposition in the correlation space.