Recalcitrant Betas: Intraday Variation in the Cross-Sectional Dispersion of Systematic Risk and Expected Returns

By

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Objective

- Study the intraday behavior of cross-sectional dispersion of asset's market betas using a large panel of high-frequency returns.

Motivation: Time-Varying Betas

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- Common approaches to account for time-varying beta:
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  - $\Rightarrow$ Time variation driven by firm characteristics
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  - $\Rightarrow$ Parametric models
- Intradaily patterns of the dispersion of market betas $\rightarrow$ This Paper

What does this paper do? (1) Econometric part

1. Derive functional limit results for the cross-sectional dispersion of beta
   One can think of it as $D^N_{t,\kappa} = \frac{1}{N}\sum_{j=1}^{N}\left(\beta^j_{t,t+\kappa} - 1\right)^2$, but formally:

$$\widehat{D}^N_{t,\kappa} = \frac{1}{N}\sum_{j=1}^{N}\left(\left(\frac{\widehat{C}^{(j)}_{t,\kappa}(2)}{\widehat{V}^{(0,j)}_{t,\kappa}}\right)^2 1_{\{\widehat{V}^{(0,j)}_{t,\kappa} > \alpha_n\}}\right), \quad \kappa \in [0,1],$$

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2. Asymptotic properties; tests; inference of the entire distribution; finite sample properties and simulation

Theorem 1. *Suppose Assumptions A and B hold. Let $n \to \infty$ and $N \to \overline{N}$, for $\overline{N} \in (0,\infty]$, with $\varpi \in (3/8, 1/2)$, $\varrho \in (0,1/2)$ and $\varrho > 2 - 4\varpi$. For $\mathcal{K}$ an arbitrary finite set of points in $[0,1]$, and $\mathcal{T}$ a finite set of positive integers in $[0,T]$, we have,*

$$\left\{\frac{\widehat{D}^N_{t,\kappa} - \widehat{B}^N_{t,\kappa} - D^N_{t,\kappa}}{\sqrt{\widehat{Avar}(\widehat{D}^N_{t,\kappa})}}\right\}_{t\in\mathcal{T},\,\kappa\in\mathcal{K}} \xrightarrow{\mathcal{L}-s} \{Z_{t,\kappa}\}_{t\in\mathcal{T},\,\kappa\in\mathcal{K}}, \tag{22}$$

where $\{Z_{t,\kappa}\}_{t\in\mathcal{T},\,\kappa\in\mathcal{K}}$ is a sequence of standard normal random variables defined on an extension of the original probability space and independent of $\mathcal{F}$.

What does this paper do? (2) Empirical part

- Empirical part: focus on temporal intradaily dispersion of market betas in the S&P 500 constituents with S&P ETF as market prices

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  4. AP implications Evidence of different monthly expected returns on strategies based on market beta sorts at market open or close

Comment #1: What I Like About the Paper

1. Research question is important and relevant
2. The derivations and proofs are very impressive
3. The empirical results are new and intriguing

Comments: (from the perspective of an asset pricing modeler)

#2 Beta decomposition

#3 Alternative interpretations

#4 What do we learn about the aggregate?

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- Why would it be useful to know? The two components might contribute differently to the downward trend at different point of time. For instance:

(1) There is empirical evidence that intraday stock return correlation increases within the day. Hence, correlation ρs (with market) are likely to have a wide dispersion at the beginning of the day.

(2) Intraday market return volatility is U shaped (quite symmetric); intraday stock return volatility is U shaped (but heavily flat towards the end).

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Suggestion:

- Writing out components (which can be toughly thought of as):

$$Var_\kappa\left(\beta^j_\kappa\right) = Var_\kappa\left(\rho^j_\kappa \times \frac{Vol^j_\kappa}{Vol^0_\kappa}\right)$$

$$= \underbrace{Cov_\kappa\left[\left(\rho^j_\kappa\right)^2, \left(\frac{Vol^j_\kappa}{Vol^0_\kappa}\right)^2\right]}_{I} + \underbrace{E_\kappa\left[\left(\rho^j_\kappa\right)^2\right] E_\kappa\left[\left(\frac{Vol^j_\kappa}{Vol^0_\kappa}\right)^2\right]}_{II} \underbrace{- \left[E_\kappa\left(\beta^j_\kappa\right)\right]^2}_{III \sim 1}$$

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- Part III will likely be about 1 during all times of the day (given the evidence in the right panel of your Figure 3 in the paper)
- Providing numerical evidence of the I (1) and II (2) decomposition will be useful. For instance, there have been different interpretations of intraday correlation and intraday volatility patterns, respectively; being able to identify which channel dominates which during which periods can help the paper relate to other literature

Comment #3: Alternative interpretations

- In Section 7, one interesting exercise is to sort days into high / medium / low VIX days (at the beginning of the day). If this is an “information flow” story (separation of different trading signals), market vol should affect the intraday beta evolution.

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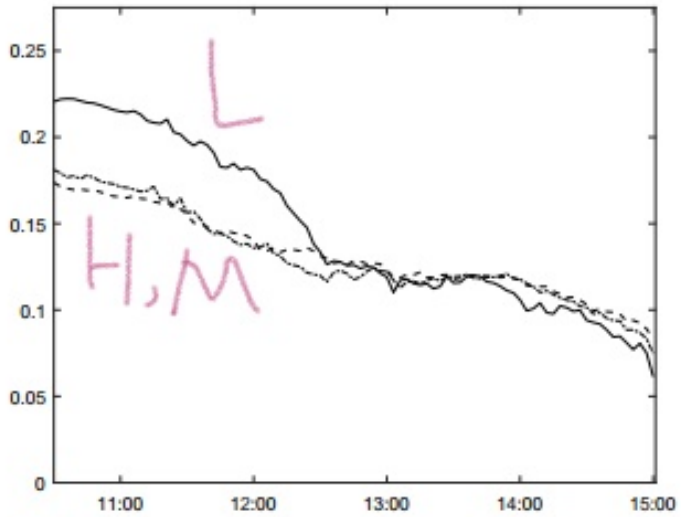

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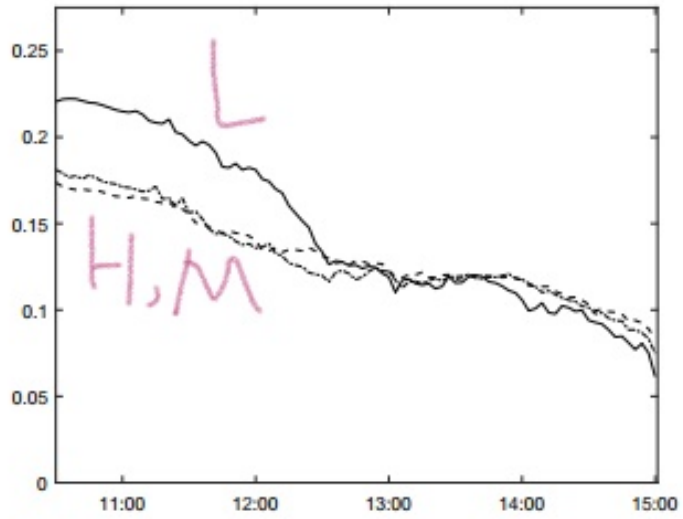
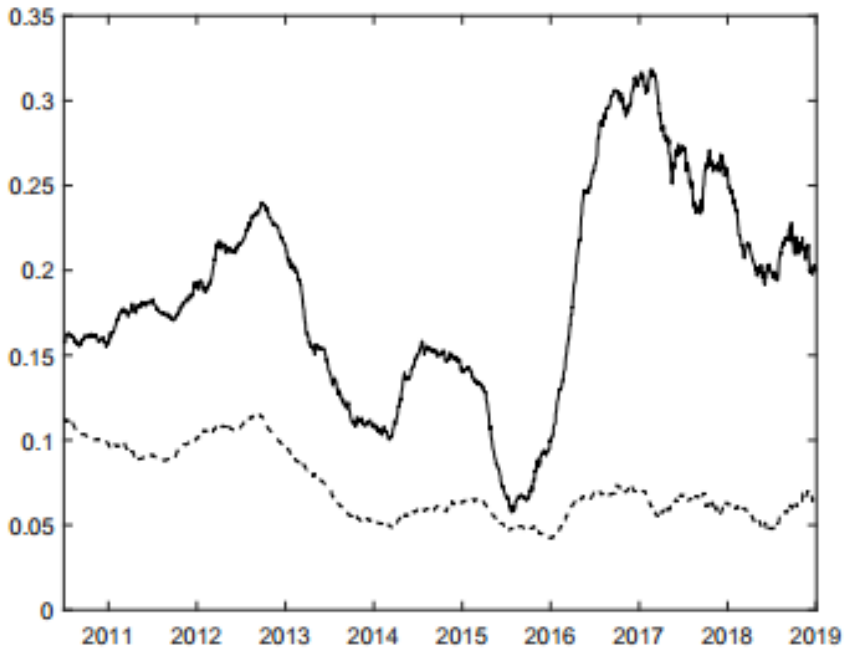


- Suggestion: Might there be a different story? I find it consistent with the Bayesian learning story (David and Veronesi) Controlling for the same amount of information, during bad times (high VIX days), investors actually take time to learn, so all assets move close to the market (“H” line lower); during good times, dispersion is always higher.

Comment #4: What do we learn at the aggregate?

- What is super interesting (also a more natural flow of the paper) is the implication for aggregate asset pricing.
- Finding: The null hypothesis of an equal intraday pattern for the cross-sectional dispersion of market betas over time is overwhelmingly rejected (Page 24) →



(A clarification question: rolling window of "past" 250 trading days?)

Comment #4: What do we learn at the aggregate?

Suggestion:

- I agree, at that point of the paper (establishing facts, pitching potential interpretations), the paper can infer some aggregate implications.
- Can we use this aggregate time series to triangulate potential interpretations as well?
  1. Relate (some versions of) OPEN DISPERSION minus CLOSE DISPERSION (previous slide) to current aggregate measures with known interpretations and have implications for asset prices/risk (e.g., DY, Amihud's liquidity measure, stock-bond comovement, VRP, or even some behavioral measures of confidence etc.)
  2. Try to predict future market excess returns or future economic growth
- Suggest dropping the beta sorts in Section 7.3 — because the paper still hasn't finalized the interpretation of beta dispersion.

Conclusion

- I highly recommend it!
- To better understand the empirical sources and interpretations of the the cross-sectional disperse in market betas,

  #2 Beta decomposition

  #3 Alternative interpretations

  #4 Aggregate implications

Thank You!

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