"Cumulant Risk Premium"

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Discussion by:

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What does this paper do?

Examines the role of cumulant risk premium (CRP) – the difference between physical and risk-neutral cumulants – in influencing single-factor linear pricing, which in turn helps come up with an estimation strategy for identifying risk premium in the capital market.

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Examines the role of cumulant risk premium (CRP) – the difference between physical and risk-neutral cumulants – in influencing single-factor linear pricing, which in turn helps come up with an estimation strategy for identifying risk premium in the capital market.

- **Theoretical model** To derive the relationship between constant-beta (leveraged) asset returns and cumulants;
- **Empirical analysis** Estimate CRP based on the previous model implications, and discuss properties at the asset and aggregate levels.

Theoretical concepts

- **Ok, before everything, what is a cumulant, and why we care**: Alternatives to moments and characteristics functions; obtained through cumulant generating functions $\kappa(t) = \log \mathrm{E}\left[e^{tX}\right]$ (which is the natural logarithm of the moment-generating function, and which in some sense is staying closer to the log-world that we care about in Finance):
  - $\Rightarrow$ $\kappa_1(X) = \mathrm{E}(X) =$ mean
  - $\Rightarrow$ $\kappa_2(X) = \mathrm{var}(X) = \mathrm{E}\left((X - \mathrm{E}(X))^2\right) =$ the variance, or second central moment.
  - $\Rightarrow$ $\kappa_3(X) = \mathrm{E}\left((X - \mathrm{E}(X))^3\right) =$ the third central moment.
  - $\Rightarrow$ $\kappa_4(X) = \mathrm{E}\left((X - \mathrm{E}(X))^4\right) - 3\left(\mathrm{E}\left((X - \mathrm{E}(X))^2\right)\right)^2 =$ the fourth central moment minus three times the square of the second central moment.
  - $\Rightarrow$ $\kappa_5(X) = \mathrm{E}\left((X - \mathrm{E}(X))^5\right) - 10\,\mathrm{E}\left((X - \mathrm{E}(X))^3\right)\mathrm{E}\left((X - \mathrm{E}(X))^2\right)$.

- **Main intuition**:
  - $\Rightarrow$ In a **lognormal** world, variance is the same in the physical and risk-neutral worlds $\rightarrow$ Therefore, the **non-linear term in asset's risk premium is zero** $\rightarrow$ Simple linear beta pricing works.
  - $\Rightarrow$ In a **non-lognormal** setting, assets are exposed not only to the variance premium (difference between physical and risk-neutral variance) but also to higher-order cumulant differences.

Empirical results

- Leveraged asset returns(w/ leverage=β) = **linear exposures β to the benchmark** + **CRP(β)**,

$$r_{ETF,t}(\beta) - rf_t = \underbrace{\beta * (r_{bmk,t} - rf_t)}_{\textbf{Linear}} + \underbrace{\alpha}_{\text{CRP}(\beta)} + \epsilon_t$$

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- **Data:** Leverage ETF with various benchmark assets (equity, bond, commodities, currencies, VIX); 2006/2007-2021; β = -3, -2, -1, 2, 3

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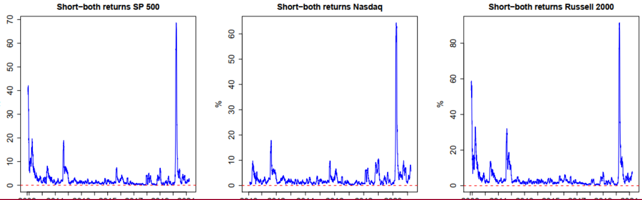
- **Data:** Leverage ETF with various benchmark assets (equity, bond, commodities, currencies, VIX); 2006/2007-2021; β = -3, -2, -1, 2, 3
- **Main results:**
  - $\Rightarrow$ CRP exists, risk-neutral $>$ physical, spikes during bad times, larger for "hedging" assets (i.e., negative β)
  - $\Rightarrow$ Average: 7-13% annualized returns in magnitude
  - $\Rightarrow$ Cross assets: stronger for equity indices, small cap stocks, financial and utilities
  - $\Rightarrow$ Construct a "short-both" trading strategy: short-selling equal amounts of two constant-β assets with opposite βs, canceling out odd-order cumulants



Nice paper!

- Provide a novel way to capture existence, magnitude, and working of non-linearity in various asset classes by exploiting a context of interesting single-factor assets
- *Coming up:* **Two extending thoughts (from an empiricist perspective)**
  1. Big picture messaging
  2. Some typical empirical quibbles

Big picture messaging

- I think about this paper as, proposing a novel method to reveal and document non-linearity in the option pricing (and hence the general capital market) that is
  1. ...informative about higher-order risk premium,
  2. ...and consistent with a closed-form theoretical solution.
- **Literature:** Then, as the paper correctly pointed out, there is a long list of papers that attempt to do the same: Bakshi, Kapadia, and Madan (2003), Backus, Chernov, and Martin (2011), Bollerslev and Todorov (2011), Martin (2013, 2017), Han and Kyle (2017), Bekaert, Engstrom, and Xu (2022) among many others
- **This paper differs in:** (A) Focusing on an asset price class that has not been explored before, and (B) exploiting the asset pricing relationship between observed log returns and CRP. The closed-form solution defines the CRP in closed-form (given the logarithm nature of returns), and hence we have the main filtered latent variable.

Big picture messaging

- Suggestions:
  1. It feels that the empirical part should come before the theoretical part, as it was unclear to a reader: (1) why we want to focus on an asset price class that can be perfectly defined with a single-factor model, with fixed beta and time-invariant factor responses? (2) what is a new moment that is being captured here that requires incorporating cumulants.
     - ⇒ Ideally, the ordering and emphases that might help the readers quickly grasp the contribution is: New empirical facts → They cannot be highly/perfectly reconciled by the existing theories proxies (based on the dynamics, the cross-section behavior, and other statistical properties of the latent variable estimates) → This calls for a new theory / concept.

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  3. For instance, I find the comparison between the Leverage ETF (positive betas) and Inverse Leverage ETF (negative betas) quite interesting (Fig.5)!

     $\Rightarrow$ CRP implied with $\beta = -3$ is always more negative (larger in magnitude) than that implied with $\beta = 3$, suggesting that investors willing to pay more to hold hedging position against the market index?

Some empirical quibbles

- Is in-sample estimation a concern here? (2020 is likely pulling most of the estimation.)
- Option / derivative market and stock market segmentation. To amplify the returns, leveraged ETFs use borrowed money to buy derivatives, such as futures contracts and option contracts. Leveraged ETFs are more complex instruments than stocks and bonds. For this reason, they tend to be used by highly-seasoned investors who understand the way they work and are willing to take the risk involved with the investment. Is that a concern? Could a higher wedge we capture simply come from heterogeneous agents rather than enlarged risk premiums?
- PC1 of short-both strategies (which nicely eliminates odd-number cumulants) is 70% correlated with VIX. I wonder if a similar short-long strategy can be designed to capture third moment, and validate using implied skewness. (Odd-number moments should also be informative about the risk premium – in terms of good risk and bad risk.)
- The “Short-both returns yen” plot in Fig.6 needs some explanations (is the lack of persistence expected?)

Conclusion

- **Highly recommend!**
- **My comments:**
  1. Big picture messaging
  2. Some typical empirical quibbles

Thank You!

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