## INTERNET APPENDICES

## A. Additional tables and figures for Section 2

Table A1: Expected realized variance

This table shows the coefficients associated with the predictors of one-month-ahead ( 22 days) total realized variance. The specifications are similar to those for realized semivariances in Table 1. The specification in column (1) assumes that the realized variance follow a Martingale $\left(E_{t}\left(r v_{t+1 m}\right)=r v_{t}\right)$. For the specifications in columns (2) to (5), we estimate the following regression setting: $E_{t}\left(r v_{t+1}\right)=$ $\hat{\alpha}+\hat{\gamma} \mathbf{G}_{\mathbf{t}}$. We consider the following predictors in matrix $\mathbf{G}$ : the total realized variance calculated over the last month $\left(r v_{t-1 m, t}\right)$; realized variance calculated using either the last five days ( $r v_{t-5 d, t}$ ) or the last day of the month $\left(r v_{t-1 d, t}\right)$; and the option-implied variance $\left(i v_{t, t+1 m}\right)$. We report, in parentheses, heteroskedasticity and autocorrelation consistent (HAC) standard deviations with 44 lags. ${ }^{* * *}$, ${ }^{* *}$, and * represent significance at the $1 \%, 5 \%$, and $10 \%$ confidence levels. The adjusted $R^{2}$ s are reported at the end of the table.

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Constant | 0 | $7.72^{* * *}$ | $7.72^{* * *}$ | $6.96^{* * *}$ | $4.15^{* * *}$ |
|  | - | $(1.28)$ | $(1.28)$ | $(1.10)$ | $(1.56)$ |
| $r v_{t-1 m, t}$ | 1 | $0.64^{* * *}$ | $0.64^{* * *}$ | $0.27^{* * *}$ | 0.12 |
|  | - | $(0.08)$ | $(0.08)$ | $(0.10)$ | $(0.09)$ |
| $r v_{t-5 d, t}$ |  |  |  | $0.32^{* *}$ | $0.29^{*}$ |
| $r v_{t-1 d, t}$ |  |  |  | $(0.16)$ | $(0.17)$ |
|  |  |  |  | $0.09^{* * *}$ | $0.06^{* *}$ |
| $i v_{t, t+1 m}$ |  |  |  | $(0.02)$ | $(0.02)$ |
| Adj. $R^{2}$ | 0.270 | 0.406 | 0.406 | 0.466 | 0.474 |

## Table A2: Correlations

This table reports correlations among the monthly U.S. downside and upside variance premiums (DVP and UVP, respectively) across various measures. Models are reported in Table 1. Panel A (Panel B) reports correlations of DVP (UVP) estimates across measures. The sample runs from April 1991 to December 2019.

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A. Correlations across models; DVP |  |  |  |  |  |
| $(1)$ | 1 |  |  |  |  |
| $(2)$ | 0.87 | 1 |  |  |  |
| $(3)$ | 0.87 | 0.99 | 1 |  |  |
| $(4)$ | 0.77 | 0.97 | 0.97 | 1 |  |
|  |  |  |  |  |  |
| $(5)$ | 0.74 | 0.97 | 0.96 | 0.99 |  |


|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| B. Correlations across models; UVP |  |  |  |  |  |
| $(1)$ | 1 |  |  |  |  |
| $(2)$ | 0.80 | 1 |  |  |  |
| $(3)$ | 0.77 | 0.94 | 1.00 |  |  |
| $(4)$ | 0.77 | 0.90 | 0.88 | 1 |  |
|  |  |  |  |  |  |
| $(5)$ | 0.77 | 0.74 | 0.75 | 0.95 |  |

Table A3: Country-level exposure and predictability patterns, time-varying exposure
This table shows the results for the following regression setting:

$$
\begin{aligned}
\kappa^{-1} r_{t, t+\kappa}^{i} & =a_{\kappa}+\left(b_{\kappa}^{D}+b_{E E, \kappa}^{D} E E_{t-1}^{i}+b_{F E, \kappa}^{D} F E_{t-1}^{i}\right) v p_{t, t+1}^{D} \\
& +\left(b_{\kappa}^{U}+b_{E E, \kappa}^{U} E E_{t-1}^{i}+b_{F E, \kappa}^{U} F E_{t-1}^{i}\right) v p_{t, t+1}^{U}+\epsilon_{i, t+\kappa}
\end{aligned}
$$

where $E E^{i}$ and $F E^{i}$ are our proxies for economic and financial exposure, respectively; they are described in Table 4 These variables are available at an annual frequency, and they are converted to monthly frequency using a step function. ${ }^{* * *},{ }^{* *}$, and * represent significance at the $1 \%, 5 \%$, and $10 \%$ confidence levels.

|  | $b_{\kappa}^{D}$ | $b_{E E, \kappa}^{D}$ | $b_{F E, \kappa}^{D}$ | $b_{\kappa}^{U}$ | $b_{E E, \kappa}^{U}$ | $b_{F E, \kappa}^{U}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa=1$ | 0.004 | 0.023 | -0.039 | $2.050^{* * *}$ | -0.092 | 0.158 | 0.836 |
|  | (0.153) | (0.018) | (0.026) | (0.488) | (0.080) | (0.158) |  |
| $\kappa=2$ | 0.197* | 0.021* | -0.035* | 0.876** | -0.115** | 0.188* | 0.570 |
|  | (0.112) | (0.012) | (0.018) | (0.383) | (0.057) | (0.099) |  |
| $\kappa=3$ | 0.205** | 0.019* | -0.023 | 1.101*** | -0.085 | 0.097 | 0.995 |
|  | (0.096) | (0.010) | (0.015) | (0.340) | (0.054) | (0.091) |  |
| $\kappa=4$ | 0.218*** | 0.019** | -0.024* | 1.363*** | -0.088* | 0.080 | 1.677 |
|  | (0.083) | (0.009) | (0.013) | (0.338) | (0.050) | (0.089) |  |
| $\kappa=5$ | 0.260*** | $0.017^{* *}$ | -0.023** | $0.977^{* * *}$ | -0.067* | 0.069 | 1.659 |
|  | (0.073) | (0.007) | (0.010) | (0.242) | (0.038) | (0.060) |  |
| $\kappa=6$ | $0.297^{* * *}$ | $0.017^{* * *}$ | $-0.024^{* * *}$ | 0.599*** | -0.065** | 0.071* | 1.789 |
|  | (0.070) | (0.006) | (0.009) | (0.204) | (0.031) | (0.043) |  |
| $\kappa=7$ | $0.288^{* * *}$ | $0.017^{* * *}$ | $-0.025^{* * *}$ | 0.285 | -0.060* | 0.087* | 1.709 |
|  | (0.068) | (0.006) | (0.008) | (0.189) | (0.032) | (0.045) |  |
| $\kappa=8$ | 0.237*** | 0.017*** | $-0.025^{* * *}$ | 0.220 | -0.058* | 0.095** | 1.361 |
|  | (0.070) | (0.006) | (0.007) | (0.186) | (0.033) | (0.041) |  |
| $\kappa=9$ | $0.217^{* * *}$ | 0.016*** | $-0.025^{* * *}$ | -0.049 | -0.044 | 0.083** | 1.189 |
|  | (0.066) | (0.006) | (0.007) | (0.191) | (0.035) | (0.041) |  |
| $\kappa=10$ | 0.193*** | 0.017*** | $-0.026^{* * *}$ | -0.125 | -0.042 | 0.086** | 1.079 |
|  | (0.063) | (0.005) | (0.006) | (0.194) | (0.031) | (0.038) |  |
| $\kappa=11$ | 0.188*** | 0.017*** | $-0.026^{* * *}$ | -0.184 | -0.040 | 0.084** | 1.158 |
|  | (0.057) | (0.004) | (0.006) | (0.190) | (0.027) | (0.035) |  |
| $\kappa=12$ | 0.181*** | $0.017^{* * *}$ | $-0.026^{* * *}$ | -0.179 | -0.045* | 0.093*** | 1.214 |
|  | (0.052) | (0.004) | (0.005) | (0.172) | (0.024) | (0.032) |  |

Table A4: Country-level exposure and predictability patterns, alternative financial exposure proxies

This table shows the results for the following regression setting:

$$
\begin{aligned}
\kappa^{-1} r_{t, t+\kappa}^{i} & =a_{\kappa}+\left(b_{\kappa}^{D}+b_{E E, \kappa}^{D} E E^{i}+b_{F E, \kappa}^{D} F E^{i}\right) v p_{t, t+1}^{D} \\
& +\left(b_{\kappa}^{U}+b_{E E, \kappa}^{U} E E^{i}+b_{F E, \kappa}^{U} F E^{i}\right) v p_{t, t+1}^{U}+\epsilon_{i, t+\kappa},
\end{aligned}
$$

where $E E^{i}$ and $F E^{i}$ are the time-series averages of our proxies for economic and financial exposure, respectively. We consider three alternative proxyes for financial exposure: the ratio of international bank claims to GDP (source: BIS, in panel A), the capital market restriction index in Fernandez et al. (2016) (panel B), and the equity market domestic investment share (source: IMF, coordinated portfolio investment survey, in panel C). ${ }^{* * *}$, ${ }^{* *}$, and * represent significance at the $1 \%, 5 \%$, and $10 \%$ confidence levels.

Panel A. International bank claims

|  | $b_{\kappa}^{D}$ | $b_{E E, \kappa}^{D}$ | $b_{F E, \kappa}^{D}$ | $b_{\kappa}^{U}$ | $b_{E E, \kappa}^{U}$ | $b_{F E, \kappa}^{U}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa=1$ | -0.039 | 0.009 | -0.001 | $2.143^{* * *}$ | 0.208 | -0.135 | 0.834 |
|  | (0.150) | (0.047) | (0.025) | (0.446) | (0.207) | (0.113) |  |
| $\kappa=2$ | 0.148 | 0.022 | -0.008 | 1.170*** | 0.041 | -0.060 | 0.524 |
|  | (0.110) | (0.033) | (0.017) | (0.372) | (0.157) | (0.084) |  |
| $\kappa=3$ | 0.170* | 0.027 | -0.010 | $1.272^{* * *}$ | -0.037 | -0.012 | 0.975 |
|  | (0.093) | (0.027) | (0.015) | (0.342) | (0.143) | (0.078) |  |
| $\kappa=4$ | 0.180** | 0.027 | -0.010 | 1.501*** | -0.056 | -0.005 | 1.637 |
|  | (0.080) | (0.023) | (0.013) | (0.342) | (0.137) | (0.076) |  |
| $\kappa=5$ | $0.220 * * *$ | 0.027 | -0.011 | $1.135^{* * *}$ | -0.060 | 0.004 | 1.616 |
|  | (0.070) | (0.019) | (0.011) | (0.250) | (0.101) | (0.056) |  |
| $\kappa=6$ | 0.255*** | 0.029* | -0.012 | $0.771^{* * *}$ | -0.089 | 0.023 | 1.707 |
|  | (0.067) | (0.016) | (0.009) | (0.205) | (0.070) | (0.039) |  |
| $\kappa=7$ | $0.245^{* * *}$ | 0.028** | -0.012 | 0.482** | -0.091 | 0.030 | 1.583 |
|  | (0.065) | (0.014) | (0.008) | (0.196) | (0.061) | (0.032) |  |
| $\kappa=8$ | $0.196 * * *$ | 0.025* | -0.010 | 0.424** | -0.072 | 0.020 | 1.208 |
|  | (0.066) | (0.014) | (0.007) | (0.199) | (0.057) | (0.029) |  |
| $\kappa=9$ | 0.179*** | 0.023* | -0.010 | 0.134 | -0.068 | 0.025 | 1.019 |
|  | (0.063) | (0.013) | (0.007) | (0.210) | (0.058) | (0.029) |  |
| $\kappa=10$ | $0.156 * * *$ | 0.021* | -0.009 | 0.063 | -0.060 | 0.021 | 0.873 |
|  | (0.060) | (0.012) | (0.006) | (0.208) | (0.056) | (0.029) |  |
| $\kappa=11$ | 0.149*** | 0.021* | -0.008 | 0.003 | -0.063 | 0.024 | 0.920 |
|  | (0.055) | (0.011) | (0.006) | (0.203) | (0.054) | (0.028) |  |
| $\kappa=12$ | $0.144^{* * *}$ | 0.020** | -0.008 | 0.015 | -0.055 | 0.018 | 0.949 |
|  | (0.051) | (0.010) | (0.006) | (0.187) | (0.049) | (0.026) |  |

Table A4: Country-level exposure and predictability patterns, alternative financial exposure proxies, continued

Panel B. Capital restrictions

|  | $b_{\kappa}^{D}$ | $b_{E E, \kappa}^{D}$ | $b_{F E, \kappa}^{D}$ |  | $b_{\kappa}^{U}$ | $b_{E E, \kappa}^{U}$ | $b_{F E, \kappa}^{U}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa=1$ | -0.068 | 0.008 | 0.033 |  | $1.846^{* * *}$ | -0.015 | 0.436 | 0.794 |
|  | $(0.163)$ | $(0.016)$ | $(0.104)$ |  | $(0.536)$ | $(0.068)$ | $(0.427)$ |  |
| $\kappa=2$ | 0.114 | 0.010 | 0.051 |  | $1.051^{* *}$ | -0.056 | 0.125 | 0.488 |
|  | $(0.119)$ | $(0.011)$ | $(0.069)$ |  | $(0.432)$ | $(0.054)$ | $(0.314)$ |  |
| $\kappa=3$ | 0.152 | 0.010 | 0.035 |  | $1.009^{* * *}$ | -0.052 | 0.301 | 0.967 |
|  | $(0.098)$ | $(0.009)$ | $(0.054)$ |  | $(0.366)$ | $(0.043)$ | $(0.274)$ |  |
| $\kappa=4$ | $0.156^{*}$ | 0.009 | 0.047 |  | $1.303^{* * *}$ | -0.058 | 0.214 | 1.637 |
|  | $(0.084)$ | $(0.007)$ | $(0.046)$ |  | $(0.368)$ | $(0.036)$ | $(0.263)$ |  |
| $\kappa=5$ | $0.188^{* *}$ | 0.008 | 0.061 |  | $1.069^{* * *}$ | $-0.050^{*}$ | 0.023 | 1.617 |
|  | $(0.073)$ | $(0.005)$ | $(0.038)$ |  | $(0.271)$ | $(0.028)$ | $(0.175)$ |  |
| $\kappa=6$ | $0.225^{* * *}$ | 0.008 | $0.059^{*}$ |  | $0.696^{* * *}$ | $-0.046^{* *}$ | 0.015 | 1.727 |
|  | $(0.069)$ | $(0.005)$ | $(0.033)$ |  | $(0.224)$ | $(0.023)$ | $(0.133)$ |  |
| $\kappa=7$ | $0.218^{* * *}$ | 0.008 | $0.055^{*}$ |  | $0.395^{*}$ | -0.037 | 0.038 | 1.616 |
|  | $(0.067)$ | $(0.005)$ | $(0.031)$ |  | $(0.209)$ | $(0.023)$ | $(0.119)$ |  |
| $\kappa=8$ | $0.168^{* *}$ | 0.007 | $0.054^{*}$ |  | $0.381^{*}$ | -0.034 | -0.012 | 1.242 |
|  | $(0.069)$ | $(0.005)$ | $(0.030)$ |  | $(0.212)$ | $(0.025)$ | $(0.116)$ |  |
| $\kappa=9$ | $0.151^{* *}$ | 0.007 | $0.052^{*}$ |  | 0.073 | -0.023 | 0.011 | 1.065 |
|  | $(0.066)$ | $(0.005)$ | $(0.027)$ |  | $(0.225)$ | $(0.026)$ | $(0.117)$ |  |
| $\kappa=10$ | $0.125^{*}$ | 0.006 | $0.054^{* *}$ |  | 0.008 | -0.021 | 0.006 | 0.933 |
|  | $(0.064)$ | $(0.004)$ | $(0.026)$ |  | $(0.234)$ | $(0.024)$ | $(0.120)$ |  |
| $\kappa=11$ | $0.119^{* *}$ | $0.007^{*}$ | $0.055^{* *}$ |  | -0.036 | -0.019 | -0.021 | 0.994 |
|  | $(0.059)$ | $(0.004)$ | $(0.023)$ |  | $(0.232)$ | $(0.021)$ | $(0.115)$ |  |
| $\kappa=12$ | $0.114^{* *}$ | $0.007^{* *}$ | $0.053^{* * *}$ |  | -0.031 | -0.021 | -0.003 | 1.023 |
|  | $(0.054)$ | $(0.003)$ | $(0.020)$ |  | $(0.212)$ | $(0.018)$ | $(0.099)$ |  |

Table A4: Country-level exposure and predictability patterns, alternative financial exposure proxies, continued

Panel C. Domestic investment share

|  | $b_{k}^{D}$ | $b_{E E, \kappa}^{D}$ | $b_{F E, \kappa}^{D}$ | $b_{\kappa}^{U}$ | $b_{E E, \kappa}^{U}$ | $b_{F E, \kappa}^{U}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa=1$ | -0.029 | 0.008 | -0.019 | 2.913** | -0.013 | -1.000 | 0.788 |
|  | (0.263) | (0.016) | (0.257) | (1.225) | (0.067) | (1.439) |  |
| $\kappa=2$ | 0.210 | 0.009 | -0.078 | 1.397 | -0.055 | -0.340 | 0.481 |
|  | (0.183) | (0.011) | (0.174) | (0.924) | (0.052) | (1.036) |  |
| $\kappa=3$ | 0.238* | 0.010 | -0.081 | 1.378** | -0.055 | -0.188 | 0.947 |
|  | (0.143) | (0.009) | (0.134) | (0.658) | (0.044) | (0.777) |  |
| $\kappa=4$ | 0.257** | 0.009 | -0.087 | 1.714*** | -0.059 | -0.336 | 1.620 |
|  | (0.125) | (0.007) | (0.115) | (0.619) | (0.036) | (0.734) |  |
| $\kappa=5$ | 0.305*** | 0.008 | -0.096 | 1.212*** | -0.049* | -0.170 | 1.599 |
|  | (0.107) | (0.006) | (0.093) | (0.429) | (0.028) | (0.504) |  |
| $\kappa=6$ | 0.348*** | 0.008 | -0.106 | 0.700* | -0.046** | 0.010 | 1.709 |
|  | (0.101) | (0.005) | (0.079) | (0.379) | (0.023) | (0.395) |  |
| $\kappa=7$ | 0.350*** | 0.008 | -0.123* | 0.290 | -0.039 | 0.181 | 1.606 |
|  | (0.097) | (0.005) | (0.071) | (0.372) | (0.024) | (0.384) |  |
| $\kappa=8$ | 0.290*** | 0.007 | -0.109 | 0.372 | -0.034 | -0.001 | 1.231 |
|  | (0.097) | (0.005) | (0.067) | (0.354) | (0.025) | (0.354) |  |
| $\kappa=9$ | 0.279*** | 0.007 | -0.119** | -0.021 | -0.024 | 0.139 | 1.058 |
|  | (0.091) | (0.005) | (0.060) | (0.349) | (0.027) | (0.323) |  |
| $\kappa=10$ | 0.251*** | 0.007* | -0.114** | -0.091 | -0.022 | 0.141 | 0.918 |
|  | (0.086) | (0.004) | (0.056) | (0.355) | (0.025) | (0.305) |  |
| $\kappa=11$ | 0.240*** | 0.007* | -0.108** | -0.151 | -0.020 | 0.136 | 0.973 |
|  | (0.078) | (0.004) | (0.052) | (0.354) | (0.022) | (0.305) |  |
| $\kappa=12$ | 0.230*** | 0.007** | -0.104** | -0.142 | -0.022 | 0.149 | 1.001 |
|  | (0.072) | (0.003) | (0.050) | (0.332) | (0.019) | (0.309) |  |



Figure A1: Alternative measures of the VP and its components
The dashed lines denote the Martingale measure, or measure (1) in Tables 1 and 2 The solid lines denote the benchmark VP measures used in the main empirical results (Table 1 and Figure 1). The shaded regions indicate NBER recessions.


Figure A2: The international stock return predictability of DVP and UVP
This figure shows the predictive coefficient estimates for the downside (DVP, top) and upside (UVP, bottom) variance premiums at horizons between one and 12 months for the main predictability regression setting:

$$
\kappa^{-1} r_{i, t, t+\kappa}=a_{i, \kappa}+a_{\kappa}+b_{\kappa}^{D} v p_{t, t+1}^{D}+b_{\kappa}^{U} v p_{t, t+1}^{U}+\epsilon_{i, t, t+\kappa},
$$

where $r_{i, t, t+\kappa}$ denotes the cumulative $\kappa$-month-ahead $\log$ excess returns for country $i$. The dashed lines depict $90 \%$ confidence intervals given Newey-West standard errors.


Figure A3: DVP coefficients, country-level regression
This figure shows the predictive coefficient estimates of the downside variance premium (the solid lines) and its $90 \%$ confidence interval given Newey-West standard errors (the dashed lines) at the country level. The regression setting is the following:

$$
\kappa^{-1} r_{i, t, t+\kappa}=a_{i, \kappa}+b_{i, \kappa}^{D} v p_{t, t+1}^{D}+b_{i, \kappa}^{U} v p_{t, t+1}^{U}+\epsilon_{i, t, t+\kappa},
$$

where $r_{i, t, t+\kappa}$ denotes the cumulative $\kappa$-month-ahead log excess returns for country $i$.
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Figure A4: UVP coefficients, country-level regression
This figure shows the predictive coefficient estimates of the upside variance premium (the solid lines) and its $90 \%$ confidence interval given Newey-West standard errors (the dashed lines) at the country level. The regression setting is the following:

$$
\kappa^{-1} r_{i, t, t+\kappa}=a_{i, \kappa}+b_{i, \kappa}^{D} v p_{t, t+1}^{D}+b_{i, \kappa}^{U} v p_{t, t+1}^{U}+\epsilon_{i, t, t+\kappa},
$$

where $r_{i, t, t+\kappa}$ denotes the cumulative $\kappa$-month-ahead log excess returns for country $i$.
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Figure A5: The international stock return predictability of U.S. and global DVP and UVP

This figure shows the predictive coefficient estimates of the downside (DVP, top) and upside (UVP, bottom) variance premiums at horizons between one and 12 months for the main predictability regression setting:

$$
\kappa^{-1} r_{i, t, t+\kappa}=a_{\kappa}+b_{\kappa}^{D} v p_{t, t+1}^{D}+b_{\kappa}^{U} v p_{t, t+1}^{U}+\epsilon_{i, t, t+\kappa},
$$

where $r_{i, t, t+\kappa}$ denotes the cumulative $\kappa$-month-ahead $\log$ excess returns for country $i$. The dashed lines depict $90 \%$ confidence intervals given Newey-West standard errors (SEs). The left panels show the coefficients when $v p^{D}$ and $v p^{D}$ are the variance premium components for the United States, while for the right panels, the variance premium components are the equalweighted average for the United States, Germany, France, and Switzerland. The sample runs from April 2003 to December 2019. Appendix Page 10

## B. A no-arbitrage international framework

This appendix complements Section 3.1 and solves a simple no-arbitrage international framework to motivate our empirical model. This framework, without loss of generality, consists of a characterization of the state evolution and a pricing kernel for a U.S./global representative agent. The U.S. state evolution process is characterized by kernel state variables, their second-moment state variables, and a cash flow state variable (dividend growth). The dynamic state process, $\boldsymbol{Y}_{\boldsymbol{t}}$, follows a VAR nature, and the shocks, $\boldsymbol{\omega}_{\boldsymbol{t}}$, are mutually independent centered gamma shocks that introduce heteroskedasticity and non-Gaussianity in an affine state variable system, as follows:

$$
\begin{align*}
Y_{t+1} & =\mu+A Y_{t}+\Sigma \omega_{t+1}  \tag{B1}\\
\omega_{t+1} & \sim \Gamma\left(\Omega Y_{t}+e, 1\right)-\left(\Omega Y_{t}+e\right)
\end{align*}
$$

where $\boldsymbol{\mu}, \boldsymbol{A}, \boldsymbol{\Sigma}, \boldsymbol{\Omega}$, and $\boldsymbol{e}$ are constant matrices; $\boldsymbol{\Gamma}$ represents a gamma distribution; $\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}$ denotes a vector of shape parameters that spans second (and higher-order) moments of these shocks; and the constant matrix $\boldsymbol{\Omega}$ describes the relative loadings. The first moment of a gamma distribution $\Gamma(x, 1)$ is $x$, and, therefore, $\boldsymbol{\Gamma}\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+e, \mathbf{1}\right)-\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)$ guarantees that the shocks $\boldsymbol{\omega}_{\boldsymbol{t + 1}}$ follow centered gamma distributions. The loading matrix $\boldsymbol{\Omega}$ can contain positive, zero, and negative coefficients; this means that a univariate process, such as real growth, can load on multiple shocks on the economy in order to realistically capture their left- and right-tail behaviors. The empirical assumption of asymmetric nonGaussian shocks allows the framework to be relatively flexible in the estimation while still keeping the model tractable, given their exponential moment-generating functions.

Next, we assume a general linear process of the log U.S. real pricing kernel, as follows:

$$
\begin{equation*}
m_{t+1}=m_{0}+\boldsymbol{m}_{\mathbf{1}} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma} \boldsymbol{\omega}_{\boldsymbol{t + \mathbf { 1 }}} \tag{B2}
\end{equation*}
$$

where $\boldsymbol{m}_{\mathbf{1}}$ and $\boldsymbol{m}_{\mathbf{2}}$ denote the loadings on the lagged state variables and the shocks, respectively.
The U.S./global investor prices individual country dividend growth processes, which load on both global and idiosyncratic kernel and cash flow shocks with heterogeneous degrees of global exposure. To be specific, we assume that dividend growth processes for the United States and country $i$ are, respectively, the following:

$$
\begin{align*}
\Delta d_{t+1} & =d_{0}+\boldsymbol{d}_{\mathbf{1}} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{d}_{\mathbf{2}} \boldsymbol{\Sigma} \boldsymbol{\omega}_{t+\mathbf{1}}  \tag{B3}\\
\Delta d_{t+1}^{i} & =d_{0}^{i}+\boldsymbol{d}_{\mathbf{1}}^{i} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{d}_{\mathbf{2}}^{i} \boldsymbol{\Sigma} \boldsymbol{\omega}_{t+\mathbf{1}}+\mu_{t}^{i}+u_{d, t+1}^{i} \tag{B4}
\end{align*}
$$

where $\boldsymbol{d}_{\mathbf{1}}^{i}\left(\boldsymbol{d}_{\mathbf{2}}^{\boldsymbol{i}}\right)$ indicates the loadings of country $i$ 's dividend growth on the U.S. lagged state variable levels (state variable shocks) and $\mu_{t}^{i}$ and $u_{d, t+1}^{i}$ indicate, respectively, the additional country-specific dividend growth mean and shock processes that are orthogonal to the U.S. shocks. Both $\boldsymbol{d}_{1}^{i}$ and $\boldsymbol{d}_{\mathbf{2}}^{i}$ can be motivated to reflect global exposure that can potentially be of an economic or financial nature.

## B.1. Solution: U.S. price-dividend ratio and log returns

Given the no-arbitrage condition, the U.S. price-dividend ratio can be rewritten as,

$$
\begin{equation*}
P D_{t}=E_{t}\left[M_{t+1}\left(\frac{P_{t+1}+D_{t+1}}{D_{t}}\right)\right]=\sum_{n=1}^{\infty} E_{t}\left[\exp \left(\sum_{j=1}^{n} m_{t+j}+\Delta d_{t+j}\right)\right] \tag{B5}
\end{equation*}
$$

where $m_{t+j}$ indicates the future $\log$ U.S. pricing kernel at month $j$ and $\Delta d_{t+j}$ the $j$-th month log dividend growth rate. Let $F_{t}^{n}$ denote the $n$-th term in the summation, $F_{t}^{n}=E_{t}\left[\exp \left(\sum_{j=1}^{n} m_{t+j}+\Delta d_{t+j}\right)\right]$, and hence $F_{t}^{n} D_{t}$ is the price of zero-coupon equity that matures in $n$ periods. The $P D_{t}$ can be rewritten as $\sum_{n=1}^{\infty} F_{t}^{n}$.

We first prove that, $\forall n \geq 1, F_{t}^{n}$ is an exactly exponential affine function of the state variables using induction. When $n=1, F_{t}^{1}=E_{t}\left[\exp \left(m_{t+1}+\Delta d_{t+1}\right)\right]=E_{t}\left\{\exp \left[\left(m_{0}+d_{0}\right)+\left(\boldsymbol{m}_{\mathbf{1}}+\boldsymbol{d}_{\mathbf{1}}\right) \boldsymbol{Y}_{\boldsymbol{t}}+\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{d}_{\mathbf{2}}\right) \boldsymbol{\Sigma} \boldsymbol{\omega}_{\boldsymbol{t}+\mathbf{1}}\right]\right\}=$ $\exp \left(e_{0}^{1}+\boldsymbol{e}_{1}^{1} \boldsymbol{Y}_{\boldsymbol{t}}\right)$, where $e_{0}^{1}$ and $\boldsymbol{e}_{1}^{1}$ are implicitly defined. Suppose that the $(n-1)$-th term $F_{t}^{n-1}=$
$\exp \left(e_{0}^{n-1}+\boldsymbol{e}_{1}^{\boldsymbol{n}-\mathbf{1}} \boldsymbol{Y}_{\boldsymbol{t}}\right)$, then

$$
\begin{align*}
F_{t}^{n} & =E_{t}\left[\exp \left(\sum_{j=1}^{n} m_{t+j}+\Delta d_{t+j}\right)\right] \\
& =E_{t}\{\exp \left(m_{t+1}+\Delta d_{t+1}\right) \underbrace{E_{t+1}\left[\exp \left(\sum_{j=1}^{n-1} m_{t+j+1}+\Delta d_{t+j+1}\right)\right]}_{F_{t+1}^{n-1}}\} \\
& =E_{t}\left[\exp \left(m_{t+1}+\Delta d_{t+1}\right) \exp \left(e_{0}^{n-1}+\boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n - 1}} \boldsymbol{Y}_{\boldsymbol{t + 1}}\right)\right]=\exp \left(e_{0}^{n}+\boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{Y}_{\boldsymbol{t}}\right) \tag{B6}
\end{align*}
$$

where $e_{0}^{n}$ and $e_{1}^{n}$ are implicitly defined. Hence, the price-dividend ratio can be solved as $P D_{t}=$ $\sum_{n=1}^{\infty} F_{t}^{n}=\sum_{n=1}^{\infty} \exp \left(e_{0}^{n}+\boldsymbol{e}_{1}^{n} \boldsymbol{Y}_{\boldsymbol{t}}\right)$. The log return can be solved with linear approximation as

$$
\begin{align*}
r_{t+1} & =\ln \left(\frac{P_{t+1}+D_{t+1}}{P_{t}}\right)=\Delta d_{t+1}+\ln \left[\frac{1+\sum_{n=1}^{\infty} \exp \left(e_{0}^{n}+\boldsymbol{e}_{\mathbf{1}}^{n} \boldsymbol{Y}_{\boldsymbol{t}+\mathbf{1}}\right)}{\sum_{n=1}^{\infty} \exp \left(e_{0}^{n}+\boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{Y}_{\boldsymbol{t}}\right)}\right] \\
& \approx \Delta d_{t+1}+\text { const. }+\frac{\sum_{n=1}^{\infty} \exp \left(e_{0}^{n}+\boldsymbol{e}_{\mathbf{1}}^{n} \overline{\boldsymbol{Y}}\right) \boldsymbol{e}_{\mathbf{1}}^{n}}{1+\sum_{n=1}^{\infty} \exp \left(e_{0}^{n}+\boldsymbol{e}_{\mathbf{1}}^{n} \overline{\boldsymbol{Y}}\right)} \boldsymbol{Y}_{\boldsymbol{t} \mathbf{1}}-\frac{\boldsymbol{e}_{\mathbf{1}}^{n}}{\sum_{n=1}^{\infty} \exp \left(e_{0}^{n}+\boldsymbol{e}_{\mathbf{1}}^{n} \overline{\boldsymbol{Y}}\right)} \boldsymbol{Y}_{\boldsymbol{t}} \\
& =\xi_{0}+\boldsymbol{\xi}_{\mathbf{1}} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma} \boldsymbol{\omega}_{\boldsymbol{t}+\mathbf{1}} . \tag{B7}
\end{align*}
$$

This produces a linear return process.

## B.2. Solution: International price-dividend ratio and log returns

The model takes the perspective of a U.S. investor. She prices country $i$ 's cash flow processes in dollars at the equilibrium. Given the common pricing kernel $m_{t+1}$, the price-dividend ratio of country $i$ is modeled as $P D_{t}^{i}=E_{t}\left[M_{t+1}\left(\frac{P_{t+1}^{i}+D_{t+1}^{i}}{D_{t}^{i}}\right)\right]=\sum_{n=1}^{\infty} E_{t}\left[\exp \left(\sum_{j=1}^{n} m_{t+j}+\Delta d_{t+j}^{i}\right)\right]$. Using similar induction procedures, it can be shown that

$$
\begin{equation*}
P D_{t}^{i}=\sum_{n=1}^{\infty} F_{t}^{n}=\sum_{n=1}^{\infty} \exp (e_{0}^{i, n}+\boldsymbol{e}_{\mathbf{1}}^{i, n} \boldsymbol{Y}_{\boldsymbol{t}}+\underbrace{e_{2}^{i, n} \boldsymbol{Y}_{t}^{i}}_{\text {Idiosyncratic Part }}) \tag{B8}
\end{equation*}
$$

where $Y_{t}^{i}$ denotes a vector of country-specific state variables. The country $i \log$ market return can be solved and approximated as,

$$
\begin{aligned}
r_{t+1}^{i} & =\ln \left(\frac{P_{t+1}^{i}+D_{t+1}^{i}}{P_{t}^{i}}\right)=\Delta d_{t+1}^{i}+\ln \left[\frac{1+\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{\mathbf{1}}^{i, n} \boldsymbol{Y}_{\boldsymbol{t}+\mathbf{1}}+\boldsymbol{e}_{\mathbf{2}}^{i, n} \boldsymbol{Y}_{\boldsymbol{t}+\mathbf{1}}^{\boldsymbol{i}}\right)}{\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{\mathbf{1}}^{i, n} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}_{\mathbf{2}}^{i, n} \boldsymbol{Y}_{\boldsymbol{t}}^{\boldsymbol{i}}\right)}\right] \\
& \approx \Delta d_{t+1}^{i}+\text { const. }+\underbrace{\frac{\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{\mathbf{1}}^{i, n} \overline{\boldsymbol{Y}}+\boldsymbol{e}_{\mathbf{2}}^{i, n} \overline{\boldsymbol{Y}}^{\boldsymbol{i}}\right) \boldsymbol{e}_{\mathbf{1}}^{i, n}}{1+\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{\mathbf{1}}^{i, n} \overline{\boldsymbol{Y}}+\boldsymbol{e}_{\mathbf{2}}^{i, n} \overline{\boldsymbol{Y}}^{i}\right)} \boldsymbol{Y}_{\boldsymbol{t + 1}}-\frac{\boldsymbol{e}_{\mathbf{1}}^{i, n}}{\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{\mathbf{1}}^{i, n} \overline{\boldsymbol{Y}}+\boldsymbol{e}_{\mathbf{2}}^{i, n} \overline{\boldsymbol{Y}}^{i}\right)} \boldsymbol{Y}_{\boldsymbol{t}}}_{\text {Global exposure }}
\end{aligned}
$$

$$
\begin{equation*}
+\underbrace{\frac{\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{1}^{i, n} \overline{\boldsymbol{Y}}+\boldsymbol{e}_{2}^{i, n} \overline{\boldsymbol{Y}}^{\boldsymbol{i}}\right) \boldsymbol{e}_{2}^{i, n}}{1+\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{1}^{i, n} \overline{\boldsymbol{Y}}+\boldsymbol{e}_{2}^{i, n} \overline{\boldsymbol{Y}}^{\boldsymbol{i}}\right)} \boldsymbol{Y}_{t+1}^{\boldsymbol{i}}-\frac{\boldsymbol{e}_{2}^{i, n}}{\sum_{n=1}^{\infty} \exp \left(e_{0}^{i, n}+\boldsymbol{e}_{1}^{i, n} \overline{\boldsymbol{Y}}+\boldsymbol{e}_{2}^{i, n} \overline{\boldsymbol{Y}}^{\boldsymbol{i}}\right)} \boldsymbol{Y}_{t}^{i}} \tag{B9}
\end{equation*}
$$

Idiosyncratic
$=\xi_{0}^{i}+\boldsymbol{\xi}_{\mathbf{1}}^{i} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{\xi}_{\mathbf{2}}^{i} \boldsymbol{\Sigma} \boldsymbol{\omega}_{\boldsymbol{t + 1}}+$ Idiosyncratic Parts.

In this framework, international stock returns are differentiated through cash flow capitalizations, where country cash flow growths are assumed with different levels of exposure to various global shocks. Intuitively, $\boldsymbol{\xi}_{2}^{i}$ is crucial in determining country $i$ 's risk premiums. Two sources of cross-country heterogeneity in $\boldsymbol{\xi}_{2}^{i}$ can be shown in a closed-form model solution, one through the pure cash flow growth $\Delta d_{t+1}^{i}$ and another through the changes in country $i$ 's log price-dividend ratio. First, we explicitly assume that country $i$ 's cash flow growth loads on global shocks through $\boldsymbol{d}_{\mathbf{2}}^{\boldsymbol{i}}$. Second, when global economic uncertainty increases, country $i$ 's future dividend growth (i.e., modeled as $\boldsymbol{d}_{\mathbf{1}}^{\boldsymbol{i}}$ ) is expected to decrease, driving down the current stock price. A bad global economic or risk aversion shock could also induce different intertemporal substitution or precautionary savings effects for different countries due to varying exposure of dividend growth to global shocks (i.e., modeled as $\boldsymbol{d}_{\mathbf{2}}^{\boldsymbol{i}}$ ), changing the interest rates and hence the total return demanded in an individual country. Therefore, both $\boldsymbol{d}_{\mathbf{1}}^{i}$ and $\boldsymbol{d}_{\mathbf{2}}^{\boldsymbol{i}}$ can enter country $i$ 's price-dividend ratio, and both can motivate heterogeneity of $\boldsymbol{\xi}_{\mathbf{2}}^{\boldsymbol{i}}$ in $r_{t+1}^{i}$.

## B.3. Solution: Variance risk premium

We derive U.S. one-period conditional stock return variances under the physical and risk-neutral expectations. First, the U.S. one-period physical conditional return variance can be easily obtained, given that $\omega_{t+\mathbf{1}} \sim \Gamma\left(\Omega Y_{t}+e, 1\right)-\left(\Omega Y_{t}+e\right)$, as

$$
\begin{equation*}
V A R_{t}\left(r_{t+1}\right)=\left(\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ 2}\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right) \tag{B10}
\end{equation*}
$$

where "o" indicates element-by-element matrix multiplication.
Second, the U.S. one-period risk-neutral conditional return variance can be obtained using the moment generating function (MGF) of gamma-distributed shocks. We start from the MGF under the risk-neutral measure

$$
\begin{aligned}
m g f_{t}^{Q}\left(r_{t+1} ; \nu\right) & =\frac{E_{t}\left[\exp \left(m_{t+1}+\nu r_{t+1}\right)\right]}{E_{t}\left[\exp \left(m_{t+1}\right)\right]} \\
& =\exp \left\{E_{t}\left(m_{t+1}\right)+\nu E_{t}\left(r_{t+1}\right)+\left[-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}-\ln \left(\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right)\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)\right\} \\
& / \exp \left\{E_{t}\left(m_{t+1}\right)+\left[-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}-\ln \left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)\right\} \\
& =\exp \left\{\nu E_{t}\left(r_{t+1}\right)+\left[-\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}+\left[-\ln \left(\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right)+\ln \left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)\right]\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)\right\}
\end{aligned}
$$

The first-order moment is the first-order derivative at $\nu=0$,

$$
\begin{aligned}
\frac{\partial m g f_{t}^{Q}\left(r_{t+1} ; \nu\right)}{\partial \nu} & =m g f_{t}^{Q}\left(r_{t+1} ; \nu\right) *\left\{E_{t}\left(r_{t+1}\right)+\left[\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma} \circ \boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma} \circ\left(\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right)^{\circ-1}\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)\right\} \\
E_{t}^{Q}\left(r_{t+1}\right) & =\left.\frac{\partial m g f_{t}^{Q}\left(r_{t+1} ; \nu\right)}{\partial \nu}\right|_{\nu=0} \\
& =E_{t}\left(r_{t+1}\right)+\left[\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma} \circ \boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma} \circ\left(1-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ-1}\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)
\end{aligned}
$$

The second-order moment can be derived as follows:

$$
\begin{aligned}
\frac{\partial^{2} m g f_{t}^{Q}\left(r_{t+1} ; \nu\right)}{\partial \nu^{2}} & =m g f_{t}^{Q}\left(r_{t+1} ; \nu\right) *\left\{E_{t}\left(r_{t+1}\right)+\left[\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma} \circ \boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma} \circ\left(\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right)^{\circ-1}\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)\right\}^{2} \\
& +m g f_{t}^{Q}\left(r_{t+1} ; \nu\right) *\left\{\left[\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma} \circ\left(\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ 2}-\left(\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right) \circ\left(\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ 2}\right] \circ\left(\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\nu} \boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right)\right. \\
E_{t}^{Q}\left(r_{t+1}^{2}\right) & =\left.\frac{\partial^{2} m g f_{t}^{Q}\left(r_{t+1} ; \nu\right)}{\partial \nu^{2}}\right|_{\nu=0} \\
& =\left(E_{t}^{Q}\left(r_{t+1}\right)\right)^{2}+\left[\left(\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ 2} \circ\left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ-2}\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right) .
\end{aligned}
$$

As a result, the one-period risk-neutral conditional variance is

$$
\begin{aligned}
V A R_{t}^{Q}\left(\widetilde{r}_{t+1}^{i}\right) & =E_{t}^{Q}\left(\left(\widetilde{r}_{t+1}^{i}\right)^{2}\right)-\left(E_{t}^{Q}\left(\widetilde{r}_{t+1}^{i}\right)\right)^{2} \\
& =\left[\left(\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ 2} \circ\left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ-2}\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)
\end{aligned}
$$

The U.S. variance risk premium, $V A R_{t}^{Q}\left(\widetilde{r}_{t+1}^{i}\right)-V A R_{t}\left(\widetilde{r}_{t+1}^{i}\right)$, is hence given by:

$$
\begin{equation*}
V A R_{t}^{Q}\left(\widetilde{r}_{t+1}^{i}\right)-V A R_{t}\left(\widetilde{r}_{t+1}^{i}\right)=\left\{\left(\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ 2} \circ\left[\left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)^{\circ-2}-\mathbf{1}\right]\right\}\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right) \tag{B11}
\end{equation*}
$$

where "o" denotes element-by-element matrix multiplication.
This provides some economic insights. First, the dynamics of VP (and its downside and upside components) should be driven by the shape parameters of kernel state variable shocks, here specified as $\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}$. This is because, for these shocks, the pricing kernel has non-zero loadings (that is, $m_{2} \neq$ 0 ). Second, for shocks with positive $m_{2}$ loadings, their shape parameters (as captured in $\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)$ ) contribute positively to VP, given $\left[\left(\frac{1}{1-m_{2} \sigma}\right)^{2}-1\right]>0$. Intuitively, for instance, in a standard habit formation model, the marginal utility loads positively on relative risk aversion, and, hence, VP in such a framework would increase with the expected variability in risk aversion; in the long-run risk model of Segal, Shaliastovich, and Yaron (2015), the kernel has a positive exposure to a bad macroeconomic shock, and, hence, VP could also increase with bad macroeconomic uncertainty.

Note that it is not trivial to derive model-implied VP components that are consistent with the downside and upside definitions as in our empirical section (negative and positive return realizations, respectively, in Section 22 because returns are endogenously determined, as shown in Equation (B7). Therefore, given the empirical focus of the paper, we choose to determine and separate the drivers of downside and upside VPs entirely empirically and let the data speak, as discussed in Section 3.1. Such an empirical approach is motivated from what we learn in this appendix section: that VP should be spanned by second moments of kernel shocks and so should its components.

## B.4. Solution: Equity risk premiums

The risk-free rate is derived as

$$
\begin{align*}
r f_{t} & =-\ln \left\{E_{t}\left[\exp \left(m_{t+1}\right)\right]\right\} \\
& =-\ln \left\{E_{t}\left(m_{t+1}\right)+\left[-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}-\ln \left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)\right\} \tag{B12}
\end{align*}
$$

We then impose the no-arbitrage condition, $1=E_{t}\left[\exp \left(m_{t+1}+r_{t+1}\right)\right]$ and obtain the expected excess returns. By expanding the law of one price equation, we obtain

$$
\begin{aligned}
1 & =E_{t}\left[\exp \left(m_{t+1}+r_{t+1}\right)\right] \\
& =\exp \left\{E_{t}\left(m_{t+1}\right)+E_{t}\left(r_{t+1}\right)+\left[-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}-\ln \left(\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right)\right]\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)\right\}
\end{aligned}
$$

where $\boldsymbol{m}_{\mathbf{2}}, \boldsymbol{\xi}_{\mathbf{2}}, \boldsymbol{\Sigma}$, and $\boldsymbol{e}$ are constant matrices defined above. Given the risk free rate derived above, the U.S. equity risk premium is given by:

$$
\begin{equation*}
E_{t}\left(r_{t+1}\right)-r f_{t}=\left\{\boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}+\ln \left[\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\xi}_{\mathbf{2}}\right) \boldsymbol{\Sigma}\right]-\ln \left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)\right\}\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right) \tag{B13}
\end{equation*}
$$

which is determined by second moments of shocks that commonly drive the pricing kernel and asset returns. Similarly, these U.S. second moments also determine the global compensation part of country $i$ 's one-month-ahead equity risk premium $\left(E P_{1, t}^{i}\right)$ in our framework, as follows:

$$
\begin{align*}
E_{t}\left(r_{t+1}^{i}\right)-r f_{t} & =\underbrace{\left\{\boldsymbol{\xi}_{\mathbf{2}}^{i} \boldsymbol{\Sigma}+\ln \left[\mathbf{1}-\left(\boldsymbol{m}_{\mathbf{2}}+\boldsymbol{\xi}_{\mathbf{2}}^{i}\right) \boldsymbol{\Sigma}\right]-\ln \left(\mathbf{1}-\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma}\right)\right\}\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)}_{\text {The Global Compensation Part }} \\
& + \text { Idiosyncratic Parts. } \tag{B14}
\end{align*}
$$

The Gaussian approximation of the US return equation above is $-\left(\boldsymbol{m}_{\mathbf{2}} \boldsymbol{\Sigma} \circ \boldsymbol{\xi}_{\mathbf{2}} \boldsymbol{\Sigma}\right)\left(\boldsymbol{\Omega} \boldsymbol{Y}_{\boldsymbol{t}}+\boldsymbol{e}\right)$, or $-\operatorname{Cov}_{t}\left(r_{t+1}, m_{t+1}\right)$; similarly, for other countries, the global part captures $-\operatorname{Cov}_{t}\left(r_{t+1}^{i}, m_{t+1}\right)$. The total country equity risk premiums can also be driven by a pure local risk compensation component, which, however, is not the focus of the paper and in theory should be unpredictable by common/U.S. predictors, and, hence, is abbreviated above without loss of generality.

In summary, this framework suggests two important implications for our research objective. First, both the dynamics of VP and the global part of international EPs should be driven by the second moments of kernel shocks. Second, this commonality implies various stock return predictability channels, which
together relate to the observed international predictive coefficients.

## C. Additional empirical evidence for Sections 3.2 and 5

## C.1. Dynamic processes

We present precise processes to estimate the equation system (12). The economic growth state variable is assumed to follow a reduced-form dynamic process that captures time-varying expected growth and asymmetric/skewed and heteroskedastic shocks to be potentially consistent with recent work (see, e.g., Adrian, Boyarchenko, and Giannone (2019)):

$$
\begin{equation*}
\theta_{t+1}=\bar{\theta}+\rho_{\theta, \theta}\left(\theta_{t}-\bar{\theta}\right)+\rho_{\theta, \theta p}\left(\theta p_{t}-\overline{\theta p}\right)+\rho_{\theta, \theta n}\left(\theta n_{t}-\overline{\theta n}\right)+\delta_{\theta, \theta p} \omega_{\theta p, t+1}-\delta_{\theta, \theta n} \omega_{\theta n, t+1} \tag{C15}
\end{equation*}
$$

where the conditional mean is subject to an $\operatorname{AR}(1)$ term capturing persistence as well as changes in expected good and bad economic uncertainties capturing the GARCH-in-mean intuition. As in Bekaert, Engstrom, and Xu (2022), the disturbance of the log economic growth is decomposed into two independent centered gamma shocks, as follows:

$$
\begin{aligned}
& \omega_{\theta p, t+1}=\Gamma\left(\theta p_{t}, 1\right)-\theta p_{t} \\
& \omega_{\theta n, t+1}=\Gamma\left(\theta n_{t}, 1\right)-\theta n_{t}
\end{aligned}
$$

where $\omega_{\theta p, t+1}\left(\omega_{\theta n, t+1}\right)$ governs the right-tail (left-tail) dynamics of the growth distribution with shape parameter $\theta p_{t}\left(\theta n_{t}\right)$ determining the conditional higher moments of the growth disturbance shock. For example, given the moment generating function (MGF) of independent gamma shocks, the conditional variance of $\theta_{t+1}$ is $\delta_{\theta, \theta p}^{2} \theta p_{t}+\delta_{\theta, \theta n}^{2} \theta n_{t}$ and the conditional unscaled skewness is $2 \delta_{\theta, \theta p}^{3} \theta p_{t}-2 \delta_{\theta, \theta n}^{3} \theta n_{t}$. Increases in $\theta p_{t}\left(\theta n_{t}\right)$ imply higher (lower) conditional skewness while increasing conditional variance, and, hence, $\theta p_{t}\left(\theta n_{t}\right)$, can be interpreted as the "good" ("bad") uncertainty state variable. This disturbance structure is one of the non-Gaussian shock assumptions that the literature has explored to realistically model macro or financial state variable processes (see, e.g., Eraker and Shaliastovich (2008); Fulop, Li, and Yu (2015); Segal, Shaliastovich, and Yaron (2015); De Groot (2015); Bekaert and Engstrom (2017); and $\mathrm{Xu}(2021)$. The dynamics of the good and bad economic uncertainty state variables follow $\mathrm{AR}(1)$ processes:

$$
\begin{align*}
\theta p_{t+1} & =\overline{\theta p}+\rho_{\theta p}\left(\theta p_{t}-\overline{\theta p}\right)+\sigma_{\theta p} \omega_{\theta p, t+1}  \tag{C16}\\
\theta n_{t+1} & =\overline{\theta n}+\rho_{\theta n}\left(\theta n_{t}-\overline{\theta n}\right)+\sigma_{\theta n} \omega_{\theta n, t+1} \tag{C17}
\end{align*}
$$

We define a macroeconomic state variable vector, $\boldsymbol{Y}_{\boldsymbol{m a c}, \boldsymbol{t}} \equiv\left[\begin{array}{lll}\theta_{t} & \theta p_{t} & \theta n_{t}\end{array}\right]^{\prime}$, and its unconditional mean $\overline{\boldsymbol{Y}_{\boldsymbol{m a c}}} \equiv\left[\begin{array}{lll}\bar{\theta} & \overline{\theta p} & \overline{\theta n}\end{array}\right]^{\prime}$. The risk aversion state variable, $q_{t}$, evolves over time with a state-dependent conditional mean and a disturbance that is exposed to fundamental economic shocks. The residual is then separated into two independent gamma shocks, $\omega_{q h, t+1}$ and $\omega_{q l, t+1}$, potentially capturing distinct behaviors of the right-tail (high risk aversion) and left-tail (low risk aversion) preference shocks:

$$
\begin{align*}
q_{t+1} & =\bar{q}+\rho_{q, q}\left(q_{t}-\bar{q}\right)+\rho_{q, q h}\left(q h_{t}-\overline{q h}\right)+\boldsymbol{\rho}_{\boldsymbol{q}, \boldsymbol{m a c}}\left(\boldsymbol{Y}_{\boldsymbol{m a c}, \boldsymbol{t}}-\overline{\boldsymbol{Y}_{\boldsymbol{m a c}}}\right) \\
& +\delta_{q, \theta p} \omega_{\theta p, t+1}+\delta_{q, \theta n} \omega_{\theta n, t+1}+\delta_{q, q h} \omega_{q h, t+1}-\delta_{q, q l} \omega_{q l, t+1}, \\
\omega_{q h, t+1} & =\Gamma\left(q h_{t}, 1\right)-q h_{t}, \\
\omega_{q l, t+1} & =\Gamma(\overline{q l}, 1)-\overline{q l}, \\
q h_{t+1} & =\overline{q h}+\rho_{q h}\left(q h_{t}-\overline{q h}\right)+\sigma_{q h} \omega_{q h, t+1} . \tag{C18}
\end{align*}
$$

The conditional mean of risk aversion evolves with the macro variables (both level and volatility), an $\mathrm{AR}(1)$ term, and a high risk aversion state variable $q h_{t}$ that captures the fluctuation of the right-tail risk aversion shock. Given that risk aversion heteroskedasticity is likely driven by its right-tail movements when risk aversion is high, we shut down heteroskedasticity coming from the left-tail movements when risk aversion is low to keep the model relatively simple. Note that our risk aversion dynamics are different from those in the literature. First, Bekaert, Engstrom, and Xu (2022) also assume a pure risk aversion shock that is orthogonal to consumption (fundamental) shocks; they assume its shape parameter is the
same as risk aversion, whereas we elicit a new state variable $q h_{t}$ that does not equal $q_{t}$ (but should very likely positively correlate with $q_{t}$ empirically, as we do find later). Second, the most acknowledged time-varying risk aversion model is Campbell and Cochrane (1999), which assumes that risk aversion is purely driven by changes in real fundamentals. Finally, we set $\boldsymbol{Y}_{\boldsymbol{q}, t}=\left[\begin{array}{ll}q_{t} & q h_{t}\end{array}\right]^{\prime}$, and $\overline{\boldsymbol{Y}_{\boldsymbol{q}}}=\left[\begin{array}{ll}\bar{q} & \overline{q h}\end{array}\right]^{\prime}$.

## C.2. Estimation results

The estimation of the three state variable system is conducted sequentially given the overlaying shocks. First, the economic growth and uncertainty state variables are estimated using a monthly sample from 1947/02 to 2019/12 and the Approximate Maximum Likelihood (AML) methodology in Bates (2006). Then, the risk aversion measure uses the $q_{t}$ series from Bekaert, Engstrom, and Xu (2022), covering from 1986/06 to 2019/12, and is first projected on known macro variables; the disturbance is estimated following Bates (2006). Below are the estimation results ( ${ }^{* * *}\left({ }^{* *},{ }^{*}\right): 1 \%(5 \%, 10 \%)$ test):

| A. Estimation Results of $\theta_{t}, \theta p_{t}, \theta n_{t}$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\theta_{t}:$ | $\bar{\theta}$ | $\rho_{\theta, \theta}$ | $\rho_{\theta, \theta p}$ | $\rho_{\theta, \theta n}$ | $\delta_{\theta, \theta p}$ | $\delta_{\theta, \theta n}$ |
| Coeff. | $0.0023^{* * *}$ | $0.3799^{* * *}$ | $4.02 \mathrm{E}-05$ | -0.0001 | $0.0001^{* * *}$ | $0.0028^{* * *}$ |
| SE | $(0.0003)$ | $(0.0313)$ | $(0.0002)$ | $(0.0012)$ | $(2.81 \mathrm{E}-5)$ | $(0.0003)$ |
| $\theta p_{t}:$ | $\overline{\theta p}$ | $\rho_{\theta p}$ | $\delta_{\theta p}$ |  |  |  |
| Coeff. | $500($ fix $)$ | $0.9979^{* * *}$ | $0.3739^{* * *}$ |  |  |  |
| SE |  | $(0.0171)$ | $(0.0173)$ |  |  |  |
| $\theta n_{t}:$ | $\overline{\theta n}$ | $\rho_{\theta n}$ | $\delta_{\theta p}$ |  |  |  |
| Coeff. | $10.3362^{* * *}$ | $0.9525^{* * *}$ | $2.2996^{* * *}$ |  |  |  |
| SE | $(2.0747)$ | $(0.0096)$ | $(0.1907)$ |  |  |  |
| B. Estimation Results of $q_{t}, q h_{t}$ |  |  |  |  |  |  |
| $q_{t}:$ | $\bar{q}$ | $\rho_{q, q}$ | $\rho_{q, q h}$ | $\rho_{q, \theta}$ | $\rho_{q, \theta p}$ | $\rho_{q, \theta n}$ |
| Coeff. | $0.3266^{* * *}$ | $0.7124^{* * *}$ | -0.0006 | $-3.1851^{* * *}$ | $0.0008^{* *}$ | 0.0011 |
| SE | $(0.0102)$ | $(0.0355)$ | $(0.0004)$ | $(0.9238)$ | $(0.0003)$ | $(0.0009)$ |
|  | $\delta_{q, \theta p}$ | $\delta_{q, \theta n}$ | $\delta_{q, q h}$ | $\delta_{q, q l}$ | $\overline{q l}$ |  |
| Coeff. | 0.0004 | $0.0185^{* * *}$ | $1.0767^{* * *}$ | $0.0906^{* * *}$ | $786.6892^{* * *}$ |  |
| SE | $(0.0003)$ | $(0.0034)$ | $(0.0645)$ | $(0.0001)$ | $(102.74)$ |  |
| $q h_{t}:$ | $\overline{q h}$ | $\rho_{q h}$ | $\delta_{q h}$ |  |  |  |
| Coeff. | $0.872^{* * *}$ | $0.5677^{* * *}$ | $1.0767^{* * *}$ |  |  |  |
| SE | $(0.0670)$ | $(0.0307)$ | $(0.0645)$ |  |  |  |

We next compare the closeness between average conditional moments (mean, variance) and empirical unconditional moments of $\theta_{t+1}$ and $q_{t+1}$. Moment matching is expected because of the highly specified model assumptions; given that our paper is not about selecting the most efficient dynamic process but obtaining realistic estimates of state variables, we do not expand the model comparison exercise and follow existing evidence and frameworks in the literature.

|  | $\theta_{t+1}$ |  | $q_{t+1}$ |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Data | Model | Data | Model |
| Mean | $0.0023^{* * *}$ | $\mathbf{0 . 0 0 2 5}$ | $0.3023^{* * *}$ | $\mathbf{0 . 3 0 4 9}$ |
|  | $(0.0003)$ |  | $(0.0084)$ |  |
| Variance | $7.33 \mathrm{E}-05^{* * *}$ | $\mathbf{6 . 1 1 E - 0 5}$ | $0.0091^{* * *}$ | $\mathbf{0 . 0 0 9 4}$ |
|  | $(7.73 \mathrm{E}-06)$ |  | $(0.0018)$ |  |

The figures below depict the dynamics of state variables in the macro and risk aversion, respectively:
(1) From top to bottom: Economic growth (gray) and its conditional mean (red); good macro uncertainty state variable $\theta p_{t}$; bad macro uncertainty state variable $\theta n_{t}$; total conditional volatility.

(2) From top to bottom: Risk aversion state variable $q_{t}$ from Bekaert, Engstrom, and Xu (2022) (gray) and its conditional mean (red); high risk aversion state variable $q h_{t}$; total conditional volatility.




## C.3. Variance decomposition of international equity risk premiums: A cross-country view

This figure complements Figure 4 (B) with a cross-country view and Figure 6 with a variance decomposition perspective. This plot shows the variance decomposition (in \%) of the model-implied international equity risk premiums at various horizons coming from different sources of state variables; by construction, at each horizon, the sum of the three numbers adds to $100 \%$. The results are calibrated using low/high economic and financial exposure, with low (high) being below the 33th (above the 67th) percentile value of the 22 countries; see construction and data details in Table 4 .


